Time Series - Shapelet/Motif Discovery



Shapelet

Time Series Classification

- Given a set X of n time series, X = {x₁, x₂, ..., x_n}, each time series has m ordered values x_i = < x_{t1}, x_{t2}, ..., x_{tm} > and a class value c_i.
- The objective is to find a function *f* that maps from the space of possible time series to the space of possible class values.
- Generally, it is assumed that all the TS have the same length *m*.

Shapelet-based Classification

8.7

7.9

4.2

3.4

- Represent a TS as a vector of distances with representative subsequences, namely shapelets.
- 2. Use it as input for machine learning classifiers.





Time Series Shapelets

- Shapelets are TS subsequences which are maximally representative of a class.
- Shapelets can provide interpretable results, which may help domain practitioners better understand their data.
- Shapelets can be significantly more accurate/robust because they are *local features*, whereas most other state-of-the-art TS classifiers consider *global features*.



Finding Shapelets

Findi	ngShapeletBF (dataset D , MAXLEN, MINLEN)
1	<i>candidates</i> ← GenerateCandidates(D , <i>MAXLEN</i> , <i>MINLEN</i>)
2	$bsf_gain \leftarrow 0$
3	For each S in candidates
4	$gain \leftarrow CheckCandidate(\mathbf{D}, S)$
5	If gain > bsf_gain
6	bsf_gain $\overleftarrow{\leftarrow}$ gain
7	$bsf_shapelet \leftarrow S$
8	EndIf
9	EndFor
10	Return bsf_shapelet

Generate Candidate

Sliding a **window of size** *l* across all of the time series objects in the dataset D, extracts all of the possible candidates and adds them to the pool

Gene	rateCandidates (dataset D , MAXLEN, MINLEN)
1	$pool \leftarrow \emptyset$
2	$l \leftarrow MAXLEN$
3	While $l \ge MINLEN$
4	For T in D
5	$pool \leftarrow pool \cup \mathbf{S}_T^{\ l}$
6	EndFor
7	$l \leftarrow l - 1$
8	EndWhile
9	Return pool











Check Candidates

Chec	kCandidate (dataset D , shapelet candidate <i>S</i>)
1	objects_histogram ← Ø
2	For each T in D
3	dist \leftarrow SubsequenceDist(T, S)
4	insert T into objects_histogram by the key dist
5	EndFor
6	Return CalculateInformationGain(<i>objects_histogram</i>)

- Inserts all of the time series objects into the histogram objects_histogram according to the distance from the time series object to the candidate
- Calculate Information Gain

Distance with a Subsequence

- Distance from the TS to the subsequence *SubsequenceDist(T, S)* is a distance function that takes time series *T* and subsequence *S* as inputs and returns a non-negative value *d*, which is the distance from *T* to *S*.
- SubsequenceDist(T, S) = min(Dist(S, S')), for $S' \in S_T^{|S|}$
 - where $S_T^{/S/}$ is the set of all possible subsequences of T
- Intuitively, it is the distance between *S* and its best matching location in *T*.



Check Candidates

Chec	ckCandidate (dataset D , shapelet candidate <i>S</i>)
1	objects_histogram ← Ø
2	For each T in D
3	dist \leftarrow SubsequenceDist(T, S)
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5	EndFor
6	Return CalculateInformationGain(<i>objects_histogram</i>)

Testing The Utility of a Candidate Shapelet

- Arrange the TSs in the dataset *D* based on the distance from the candidate.
- Find the optimal split point that maximizes the information gain (same as for Decision Tree classifiers)
- Pick the candidate achieving best utility as the shapelet



Entropy



- A TS dataset D consists of two classes, A and B.
- Given that the proportion of objects in class A is p(A) and the proportion of objects in class B is p(B),
- The **Entropy** of D is: I(D) = -p(A)log(p(A)) p(B)log(p(B)).
- Given a strategy that divides D into two subsets D_1 and D_2 , the information remaining in the dataset after splitting is defined by the weighted average entropy of each subset.
- If the fraction of objects in D_1 is $f(D_1)$ and in D_2 is $f(D_2)$, the total entropy of D after splitting is $\hat{I}(D) = f(D_1)I(D_1) + f(D_2)I(D_2)$.

Information Gain



Split point

distance from

shapelet = 5.1

- Given a certain split strategy *sp* which divides D into two subsets D_1 and D_2 , the entropy before and after splitting is I(D) and $\hat{I}(D)$.
- The **information gain** for this splitting rule is:

•
$$Gain(sp) = I(D) - \hat{I}(D) =$$

= $I(D) - f(D_1)I(D_1) + f(D_2)I(D_2).$

• We use the distance from *T* to a shapelet *S* as the splitting rule *sp*.

Problem

• The total number of candidate is

 $\sum_{l=MINLEN}^{MAXLEN} \sum_{T_i \in D} (|T_i| - l + 1)$

- For each candidate you have to compute the distance between this candidate and each training sample (space inefficiency)
- For instance
 - 200 instances with length 275
 - 7,480,200 shapelet candidates

Speedup

- Distance calculations form TSs to shapelet candidates is expensive.
- Reduce the time in two ways
 - Distance Early Abandon: reducing the distance computation time between two TS
 - Admissible Entropy Pruning: reducing the number of distance calculations



Distance Early Abandon

- We only need the minimum distance.
- Method:
 - Keep the best-so-far distance
 - Abandon the calculation if the partial current distance is larger than best-so-far.
 - We can avoid to compute the full distance for S if the partial one is greater than the best so far



Admissible Entropy Pruning

- We only need the best shapelet for each class
- For a candidate shapelet
 - We do not need to calculate the distance for each training sample
 - After calculating some training samples, the **upper bound** of information gain (corresponding to the optimistic scenario) < best candidate shapelet
 - Stop calculation for that candidate and try next candidate



Motif

Time Series Motif Discovery

- Finding repeated patterns, i.e., pattern mining.
- Are there any repeated patterns, of length *m* in the TS?



Why Find Motifs?

- Mining association rules in TS requires the discovery of motifs. These are referred to as primitive shapes and frequent patterns.
- Several TS classifiers work by constructing typical prototypes of each class. These prototypes may be considered motifs.
- Many TS anomaly detection algorithms consist of modeling normal behavior with a set of typical shapes (which we see as motifs), and detecting future patterns that are dissimilar to all typical shapes.

- The Matrix Profile (MP) is a data structure that annotates a TS and can be exploited for many purposed: e.g. efficient Motif Discovery.
- Given a time series, T and a desired subsequence length, m.





m



|**T**|-**m**+1

 $\ \ \sim$

0	7.6952	7.7399	•••
7.6952	0	7.7106	•••
7.7399	7.7106	0	•••
•••	•••		

 For each subsequence we keep only the distance with the closest nearest neighbor. set of corresponding



• The **distance** to the corresponding **nearest neighbor** of each subsequence can be stored in a vector called **matrix profile P**.



The matrix profile value at location i is the distance between T_i and its nearest neighbor

• The **index of corresponding nearest neighbor** of each subsequence is also stored in a vector called **matrix profile index**.



The matrix profile value at location i is the distance between T_i and its nearest neighbor

- The MP index allows to find the nearest neighbor to any subsequence in constant time.
- Note that the pointers in the matrix profile index are not necessarily symmetric.
- If A points to B, then B may or may not point to A
- The classic TS motif: the two smallest values in the MP must have the same value, and their pointers must be mutual.



How to "read" a Matrix Profile

- For relatively low values, you know that the subsequence in the original TS must have (at least one) relatively similar subsequence elsewhere in the data (such regions are "motifs")
- For relatively high values, you know that the subsequence in the original TS must be unique in its shape (such areas are anomalies).



How to Compute Matrix Profile?

• Given a time series, T and a desired subsequence length, m.



	inf																		
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Matrix profile is initialized as inf vector

This is just a toy example, so the values and the vector length does not fit the time series shown above

How to Compute Matrix Profile?

• Given a time series, T and a desired subsequence length, m.



| inf |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | | | | | | | | | | | | | | | | | | |

At the first iteration, a subsequence T_i is randomly selected from T

How to Compute Matrix Profile?

• Given a time series, T and a desired subsequence length, m.



| inf |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | | | | | | | | | | | | | | | | | | 1 |

We compute the distances between T_i and every subsequences from T (time complexity = O(|T|log(|T|))) We then put the distances in a vector based on the position of the subsequences

3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1

The distance between T_i and T_1 (first subsequence) is 3
• Given a time series, T and a desired subsequence length, m.

inf]																		
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We compute the distances between T_i and every subsequences from T (time complexity = O(|T|log(|T|))) We them put the distances in a vector based on the position of the subsequences

Let say T_i happen to be the third subsequences, therefore the third value in the distance vector is 0



	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
mi	min Matrix profile is updated by apply elementwise minimum to these two vectors																		
	3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1



3	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf	inf
mi	min these two vectors																	
3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1



3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1

After we finish to update matrix profile for the first iteration

• Given a time series, T and a desired subsequence length, m.



In the second iteration, we randomly select another subsequence T_j and it happens to be the 12^{th} subsequences

• Given a time series, T and a desired subsequence length, m.



Once again, we compute the distance between T_i and every subsequences of T

2	3	1	4	4	3	6	2	1	5	8	0	2	3	5	9	4	2	2]
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---











• Given a time series, T and a desired subsequence length, m.



We repeat the two steps (distance computation and update) until we have used every subsequences

• Given a time series, T and a desired subsequence length, m.



There are $|\mathsf{T}|$ subsequences and the distance computation is $O(|\mathsf{T}|\log(|\mathsf{T}|))$

The overall time complexity is $O(|T|^2 \log(|T|))$

Motif Discovery From Matrix Profile



Local minimums are corresponding to motifs

Motif Discovery From Matrix Profile



 It is sometime useful to think of time series subsequences as points in m-dimensional space.

 In this view, dense regions in the m-dimensional space correspond to regions of the time series that have a low corresponding MP.



Motif/Shapelet Summary

• A **motif** is a repeated pattern/subsequence in a given TS.

• A **shapelet** is a pattern/subsequence which is maximally representative of a class with respect to a given dataset of TSs.



References

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Matrix Profile I: All Pairs Similarity Joins for Time Series: A Unifying View that Includes Motifs, Discords and Shapelets

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Advances. The of-physical-inderivatives of a standing of the s ine and early aband loning) at best produ It is exact, providing no false positives or false dismis ale speedup. In this work we introduce It is simple and parameter-free. In contrast, the mo general metric space APSS algorithms require building and tuning spatial access methods and/or hash function time. The ex Our algorithm requires an inconsequential space overhead y join algorithm computes the answer to the m (a) with a small constant factor if and time series discord problem as a vide offer, and ou orithm incidentally provides the factors known algorithm for these extensively studied problems. We demonstrate the ity of our ideas for many time series data mining problems While our exact algorithm is extremely extremely large datasets we can compute the results in a anytime fashion, allowing ultra-fast approximate solution segmentation, density estimation, and contrast set incrementally update if very entreasony, an analysis of this means we can effectively maintain exact joins on streaming data forever. Our method provides *full* joins, eliminating the need to specify a similarity threshold, which as we will show is a near impossible task in this domain.

Keywords-Time Series; Similarity Joins; Motif Discover I INTRODUCTION

ilarity-search (also known as Our algorithm is embarrassingly parallelizable, both on multi-our resources and in distributed costeme omes in several variants. The basic task is this f data objects, retrieve the nearest neighbor In the text domain the algorithm has

We believe that this lack of progress stems not from a lack nterest in this useful primitive, but from the daunting nature ider the following en ds of an industrial collaborator. A boiler at a chemi ice a minute. After a year, we hav series of length 525,600. A plant manager may wish to d rity self-join on this data with week-long subs to discover operating regimes (summer vs. winter or stillate vs. heavy distillate etc.) The obvious nested loop mr requires 132,880,692,960 Euclidean distance rk is to show that we can reduce this t off-the-shelf desktop computer. More ted and/or undated incre ain this join essentially forever on a standard

Dept. of Computer Science & Engineering University of California, Riverside, CA 92521 lexiangy@cs.ucr.edu Dept. of Computer Science & Engineering University of California, Riverside, CA 9252 eamonn@cs.ucr.edu ome time to consider a detailed mor

Classification of time series has been attracting great interest over the past decade. Recent empirical evidence has strongly ungested that the simple mearest neighbor algorithm is very difficult to beat for most time series problems. While this may be considered good ishows some examples of leaves from two classes. Urica dioic (stinging nettles) and Varbera articifolia. These two plants in ommonly confused, hence the colloquial name "false . arbona articifolia. news, given the simplicity of implementing the nearest neighalgorithm, there are some negative conseq ces of this. First, th earest neighbor algorithm requires storing and searching th ire dataset, resulting in a time and space com

ae series shapelet primitives can be interpretable, more

INTRODUCTION While the last decode has seen a large interest in time series classification, to date the most accurate and robust method is the simple asserst neighbor algorithm [4][12][14]. While the searcet and accurate has the accurates of simplicity and not series and the second seco

and significantly faster than state-of-the-art classifier

Categories and Subject Descriptors H.2.8 [Database Management]: Database Applications

Lexiang Ye

ABSTRACT

n this work we introduc

General Terms Algorithms, Experime

1. INTRODUCTION

mple nearest neighbor algorithm righbor algorithm has the adva quiring extensive parameter t

Time Series Shapelets: A New Primitive for Data Mining

Figure 1: Samples of leaves from two species. Note that leaves have the invect-bits damage uppose we wish to build a classifier to di

Eamonn Keogh

olor and size within each class completely dwarfs the int ommon for leaves to have distortions or "occlusions" due to nsect damage, and these are likely to confuse any globa neasures of shape. Instead we attempt the following. We first suvert each leaf into a one-dimensional re-

arements, and the fact that it does not tell us anything about a particular object was assigned to a particular class. this work we present a novel time series data mining pri led time series shapelets. Informally, shapelets are time serie sequences which are in some sense maximally representativ a class. While we believe shapelets can have many uses in dat ng, one obvious implication of them is to mitigate the two es of the nearest neighbor algorithm noted abo

orter tuning it does have a

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Figure 2: A shape can be a series" representation. The i time series will be made app

ears [8]. However, here we find that using a nearest lassifier with either the (rotation invariant) Euch ynamic Time Warping (DTW) distance does atperform readom guessing. The reason

to be due to the fact that the data is somewhat noisy (i.e. mee bites, and different stem lengths), and this noise is enough t rame the subtle differences in the shape

