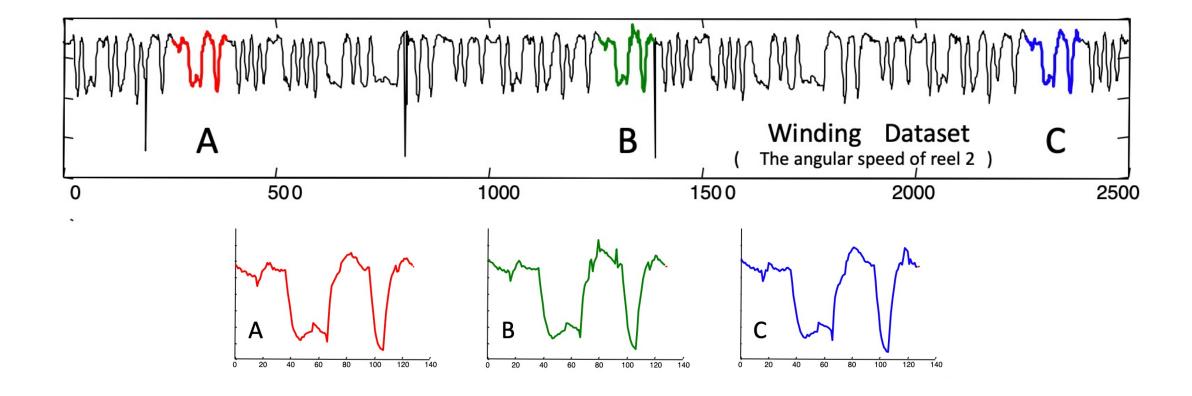
## Time Series - Shapelet/Motif Discovery



# Motif

#### Time Series Motif Discovery

- Finding repeated patterns, i.e., pattern mining.
- Are there any repeated patterns, of length *m* in the TS?

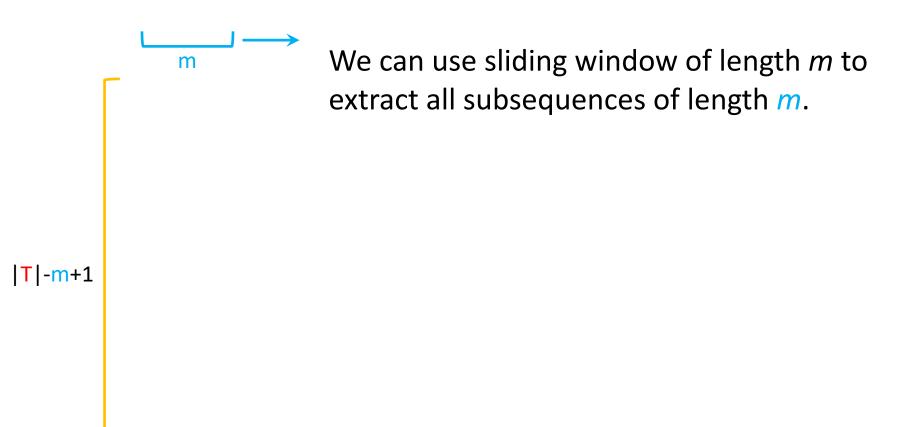


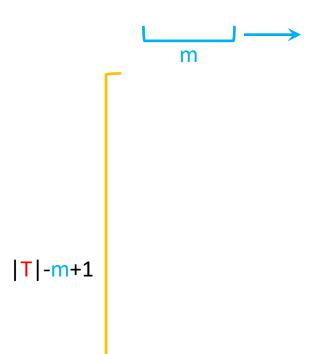
## Why Find Motifs?

- Mining association rules in TS requires the discovery of motifs. These are referred to as primitive shapes and frequent patterns.
- Several **TS classifiers** work by constructing typical prototypes of each class. These prototypes may be considered motifs.
- Many **TS** anomaly detection algorithms consist of modeling normal behavior with a set of typical shapes (which we see as motifs), and detecting future patterns that are dissimilar to all typical shapes.

- The Matrix Profile (MP) is a data structure that annotates a TS and can be exploited for many purposed: e.g. efficient Motif Discovery.
- Given a time series, T and a desired subsequence length, m.

m





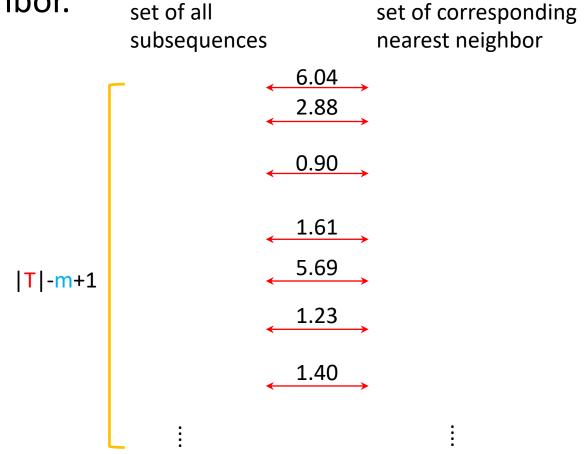
We can then compute the pairwise distance among these subsequences.

0	7.6952	7.7399	•••
7.6952	0	7.7106	•••
7.7399	7.7106	0	

...

• For each subsequence we keep only the distance with the closest nearest neighbor.

set of corresponding

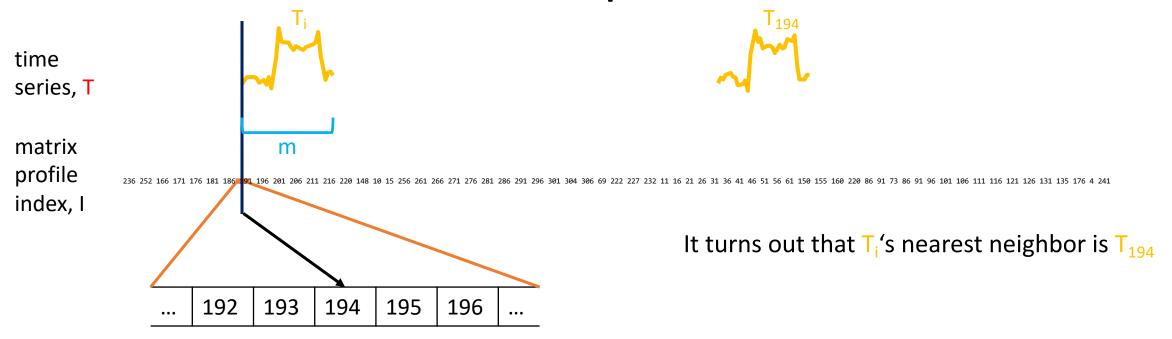


• The distance to the corresponding nearest neighbor of each subsequence can be stored in a vector called matrix profile P.



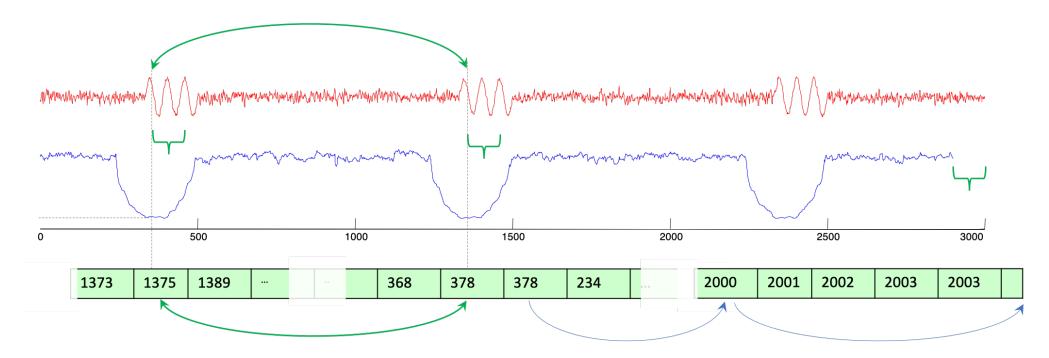
The matrix profile value at location i is the distance between  $T_i$  and its nearest neighbor

• The index of corresponding nearest neighbor of each subsequence is also stored in a vector called matrix profile index.



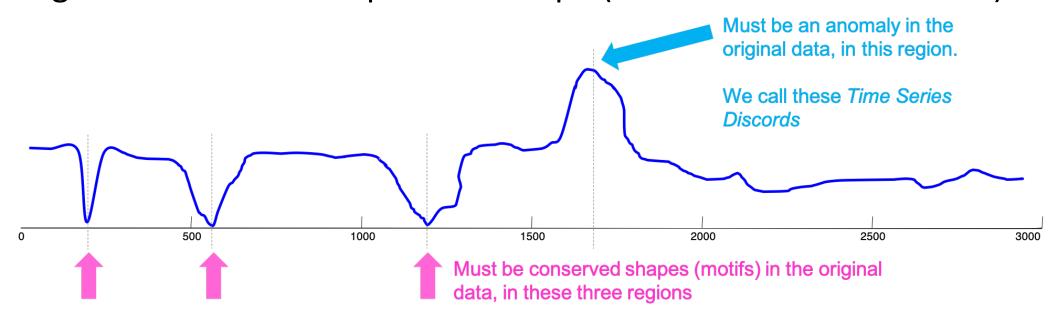
The matrix profile value at location i is the distance between  $T_i$  and its nearest neighbor

- The MP index allows to find the nearest neighbor to any subsequence in constant time.
- Note that the pointers in the matrix profile index are not necessarily symmetric.
- If A points to B, then B may or may not point to A
- The classic TS motif: the two smallest values in the MP must have the same value, and their pointers must be mutual.



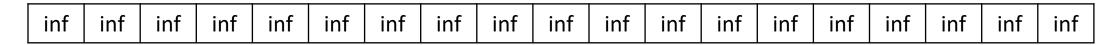
#### How to "read" a Matrix Profile

- For relatively low values, you know that the subsequence in the original TS must have (at least one) relatively similar subsequence elsewhere in the data (such regions are "motifs")
- For relatively high values, you know that the subsequence in the original TS must be unique in its shape (such areas are anomalies).



Given a time series, T and a desired subsequence length, m.





Matrix profile is initialized as inf vector

This is just a toy example, so the values and the vector length does not fit the time series shown above

Given a time series, T and a desired subsequence length, m.



		inf																		
--	--	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

At the first iteration, a subsequence  $T_i$  is randomly selected from T

Given a time series, T and a desired subsequence length, m.



We compute the distances between  $T_i$  and every subsequences from T (time complexity =  $O(|T|\log(|T|))$ ) We then put the distances in a vector based on the position of the subsequences

3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1
	l			l											l			1



Given a time series, T and a desired subsequence length, m.



m

| inf |
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

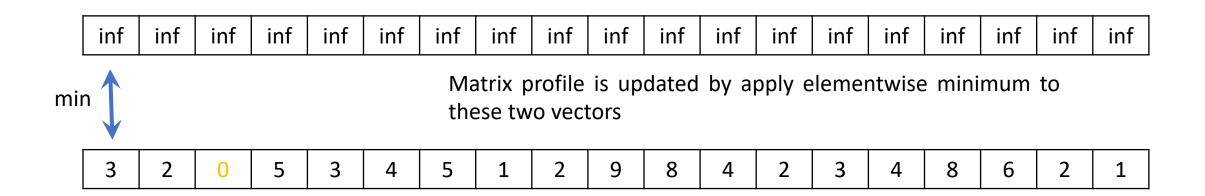
We compute the distances between  $T_i$  and every subsequences from T (time complexity =  $O(|T|\log(|T|))$ ) We them put the distances in a vector based on the position of the subsequences

3	2 0 5	3 4	5 1 2	9 8 4	2 3	4 8 6	2
---	-------	-----	-------	-------	-----	-------	---

Let say  $T_i$  happen to be the third subsequences, therefore the third value in the distance vector is 0

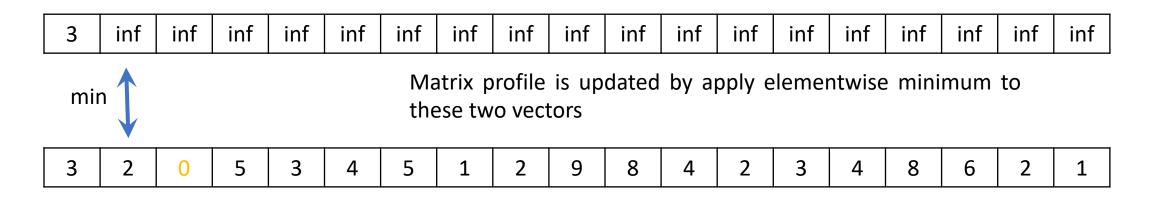
Given a time series, T and a desired subsequence length, m.





Given a time series, T and a desired subsequence length, m.





Given a time series, T and a desired subsequence length, m.



3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1
	1			1														1 1

After we finish to update matrix profile for the first iteration

3	2	0	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Given a time series, T and a desired subsequence length, m.



m

	3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1
- 1																			

In the second iteration, we randomly select another subsequence  $T_j$  and it happens to be the  $12^{th}$  subsequences

• Given a time series, T and a desired subsequence length, m.



m

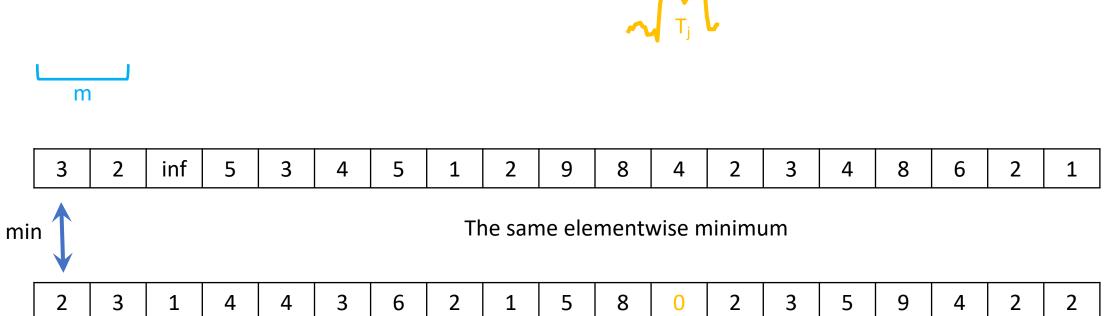
3	2	inf	5	3	4	5	1	2	9	8	4	2	3	4	8	6	2	1	
---	---	-----	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	--

Once again, we compute the distance between T<sub>i</sub> and every subsequences of T

2	3	1	4	4	3	6	2	1	5	8	0	2	3	5	9	4	2	2
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

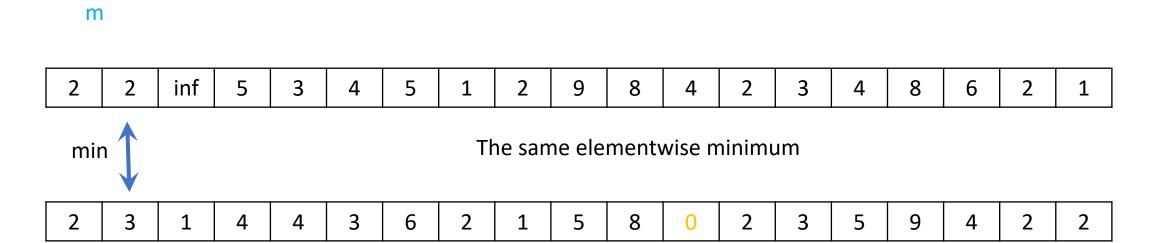
Given a time series, T and a desired subsequence length, m.





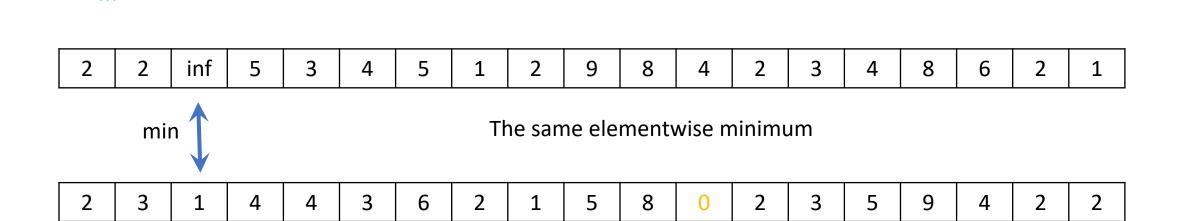
• Given a time series, T and a desired subsequence length, m.





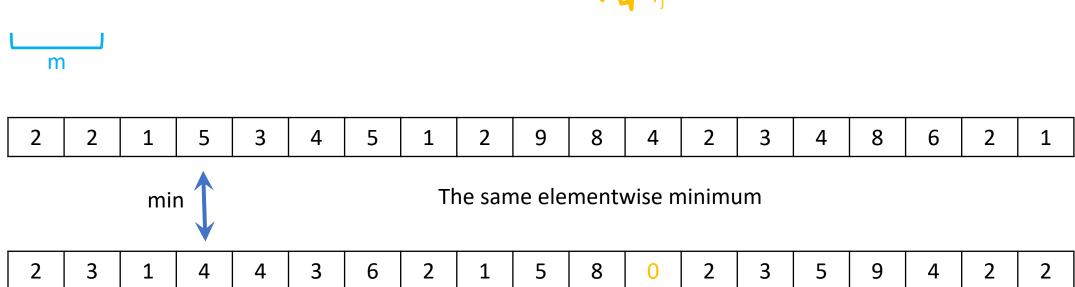
Given a time series, T and a desired subsequence length, m.



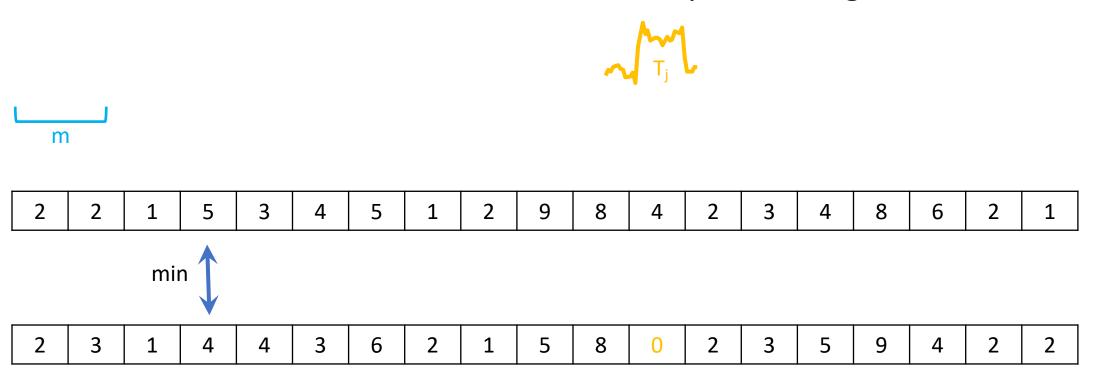


Given a time series, T and a desired subsequence length, m.





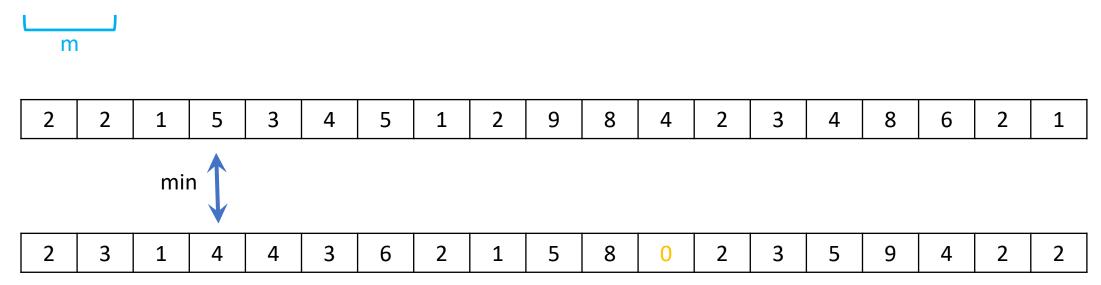
• Given a time series, T and a desired subsequence length, m.



We repeat the two steps (distance computation and update) until we have used every subsequences

Given a time series, T and a desired subsequence length, m.

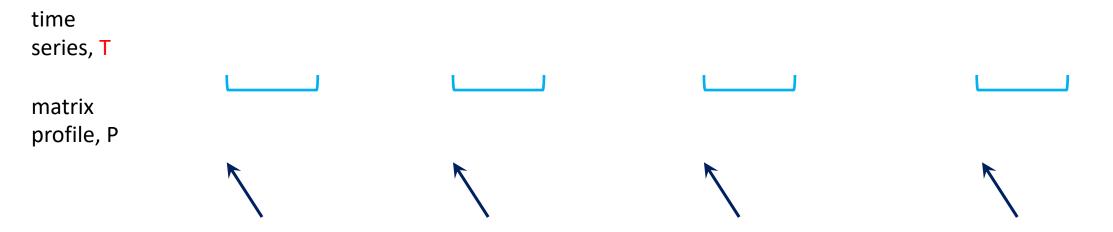




There are |T| subsequences and the distance computation is  $O(|T|\log(|T|))$ 

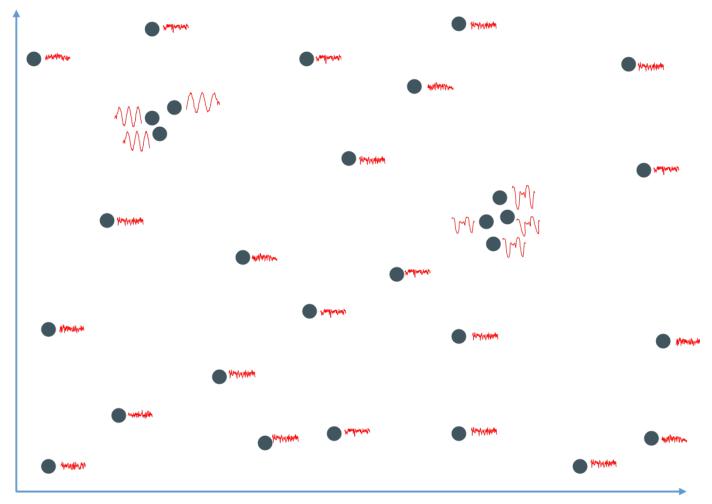
The overall time complexity is  $O(|T|^2 \log(|T|))$ 

#### Motif Discovery From Matrix Profile



Local minimums are corresponding to motifs

## Motif Discovery From Matrix Profile



- It is sometime useful to think of time series subsequences as points in m-dimensional space.
- In this view, dense regions in the m-dimensional space correspond to regions of the time series that have a low corresponding MP.

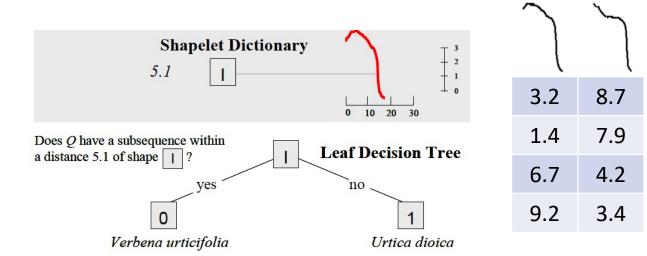
# Shapelet

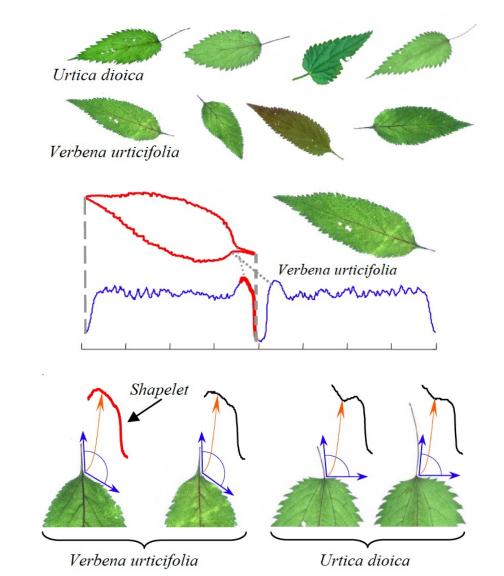
#### Time Series Classification

- Given a set X of n time series,  $X = \{x_1, x_2, ..., x_n\}$ , each time series has m ordered values  $x_i = \langle x_{t1}, x_{t2}, ..., x_{tm} \rangle$  and a class value  $c_i$ .
- The objective is to find a function f that maps from the space of possible time series to the space of possible class values.
- Generally, it is assumed that all the TS have the same length m.

#### Shapelet-based Classification

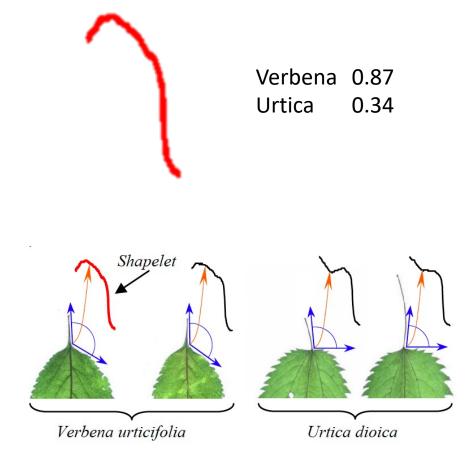
- 1. Represent a TS as a vector of distances with representative subsequences, namely shapelets.
- 2. Use it as input for machine learning classifiers.





#### Time Series Shapelets

- Shapelets are TS subsequences which are maximally representative of a class.
- Shapelets can provide interpretable results, which may help domain practitioners better understand their data.
- Shapelets can be significantly more accurate/robust because they are *local features*, whereas most other state-of-theart TS classifiers consider *global features*.



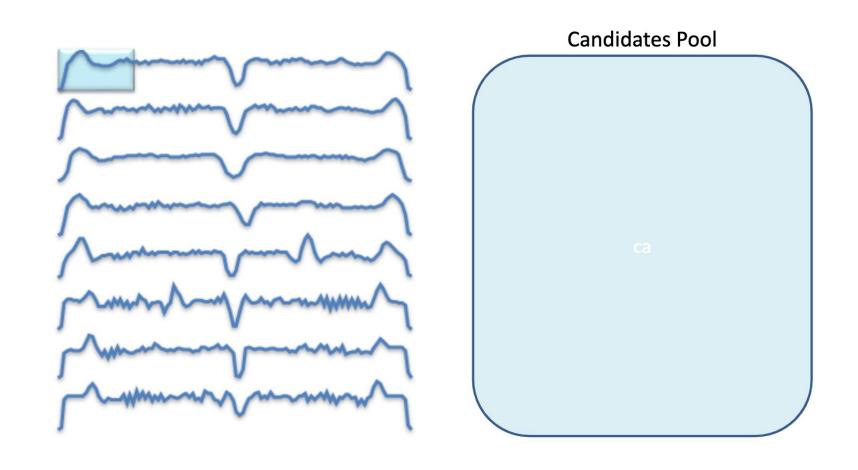
## Finding Shapelets

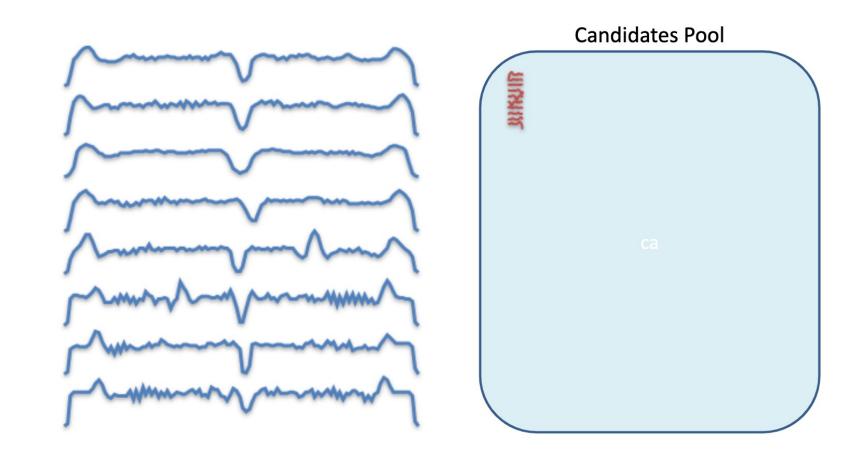
```
FindingShapeletBF (dataset D, MAXLEN, MINLEN)
       candidates 	← GenerateCandidates(D, MAXLEN, MINLEN)
       bsf gain \leftarrow 0
       For each S in candidates
4
5
           gain \leftarrow CheckCandidate(D, S)
           If gain > bsf gain
6
               bsf gain \leftarrow gain
               bsf shapelet \leftarrow S
7
8
9
           EndIf
       EndFor
       Return bsf shapelet
```

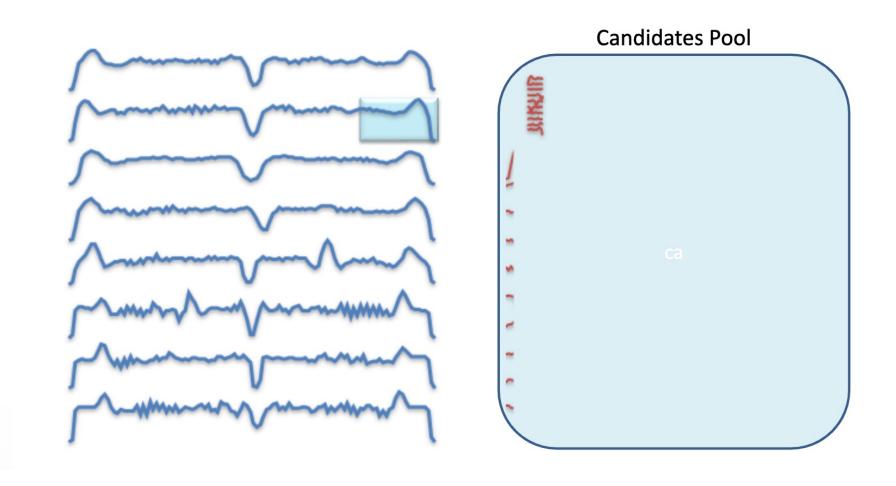
#### **Generate Candidate**

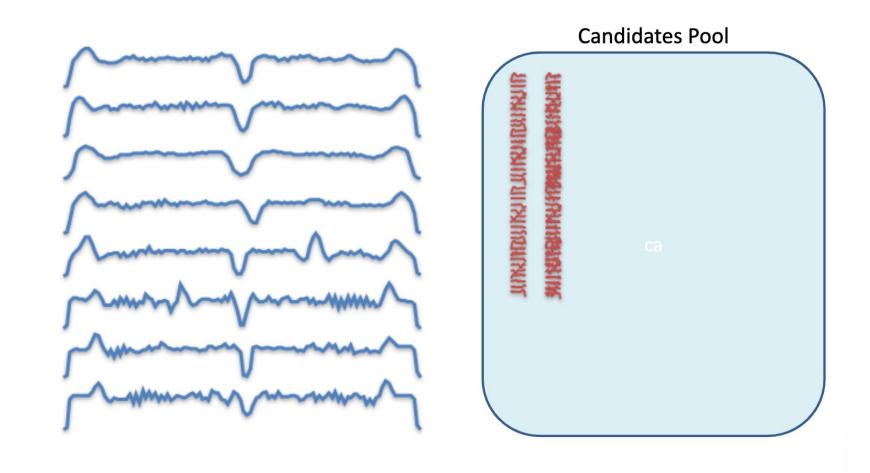
Sliding a **window of size** *l* across all of the time series objects in the dataset D, extracts all of the possible candidates and adds them to the pool

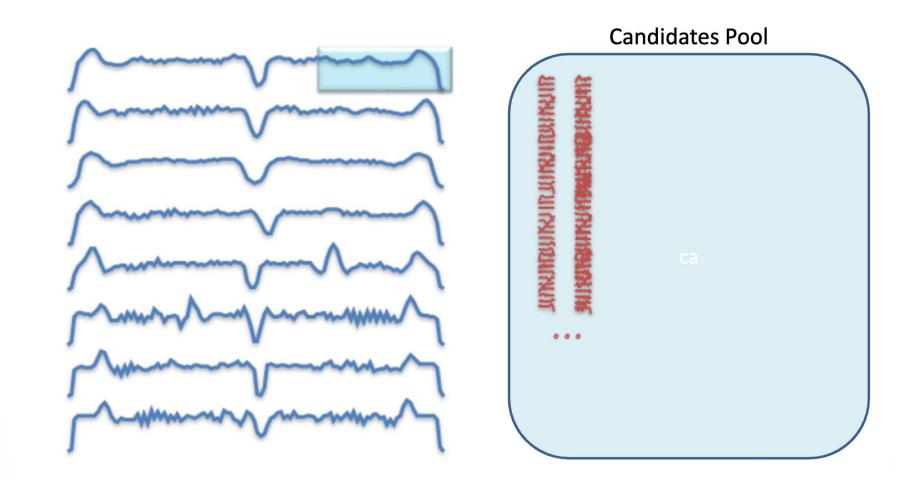
#### Extract Subsequences of all Possible Lengths









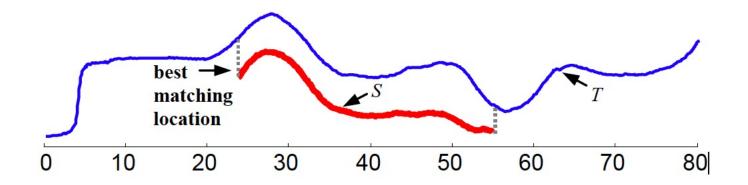


## **Check Candidates**

CheckCandidate (dataset <b>D</b> , shapelet candidate S)	
1	objects_histogram ← Ø
2	For each T in D
3	$dist \leftarrow SubsequenceDist(T, S)$
4	insert T into objects_histogram by the key dist
5	EndFor
6	Return CalculateInformationGain(objects_histogram)

## Distance with a Subsequence

- Distance from the TS to the subsequence SubsequenceDist(T, S) is a distance function that takes time series T and subsequence S as inputs and returns a nonnegative value d, which is the distance from T to S.
- SubsequenceDist(T, S) = min(Dist(S, S')), for S'  $\in S_T^{|S|}$
- where  $S_T^{|S|}$  is the set of all possible subsequences of T
- Intuitively, it is the distance between S and its best matching location in T.

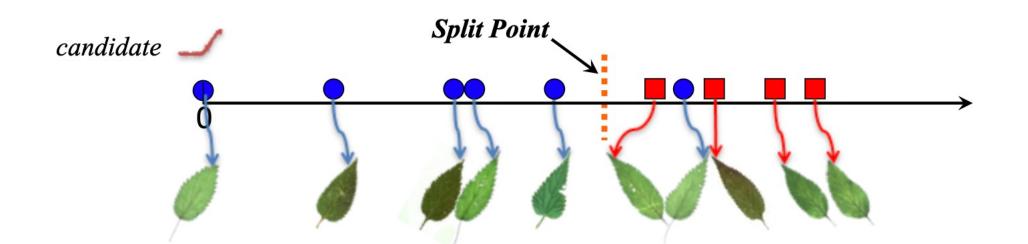


## **Check Candidates**

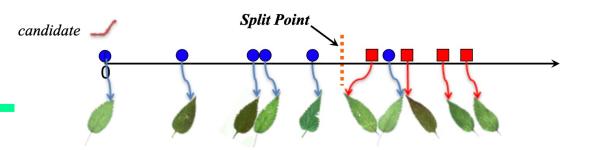
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4	insert T into objects_histogram by the key dist
5	EndFor
6	Return CalculateInformationGain(objects_histogram)

# Testing The Utility of a Candidate Shapelet

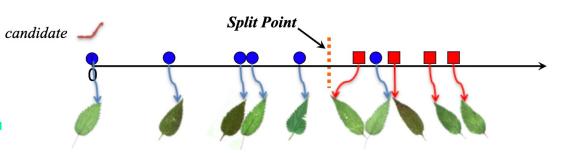
- Arrange the TSs in the dataset D based on the distance from the candidate.
- Find the optimal split point that maximizes the information gain (same as for Decision Tree classifiers)
- Pick the candidate achieving best utility as the shapelet



### Entropy



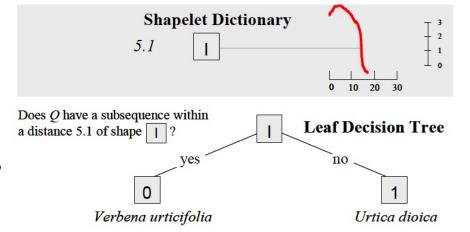
- A TS dataset D consists of two classes, A and B.
- Given that the proportion of objects in class A is p(A) and the proportion of objects in class B is p(B),
- The **Entropy** of D is: I(D) = -p(A)log(p(A)) p(B)log(p(B)).
- Given a strategy that divides the D into two subsets  $D_1$  and  $D_2$ , the information remaining in the dataset after splitting is defined by the weighted average entropy of each subset.
- If the fraction of objects in  $D_1$  is  $f(D_1)$  and in  $D_2$  is  $f(D_2)$ ,
- The total entropy of D after splitting is  $\hat{I}(D) = f(D_1)I(D_1) + f(D_2)I(D_2)$ .



### Information Gain

- Given a certain split strategy sp which divides D into two subsets  $D_1$  and  $D_2$ , the entropy before and after splitting is I(D) and  $\hat{I}(D)$ .
- The **information gain** for this splitting rule is:
- $Gain(sp) = I(D) \hat{I}(D) =$
- =  $I(D) f(D_1)I(D_1) + f(D_2)I(D_2)$ .

 We use the distance from T to a shapelet S as the splitting rule sp. Split point distance from shapelet = 5.1



### Problem

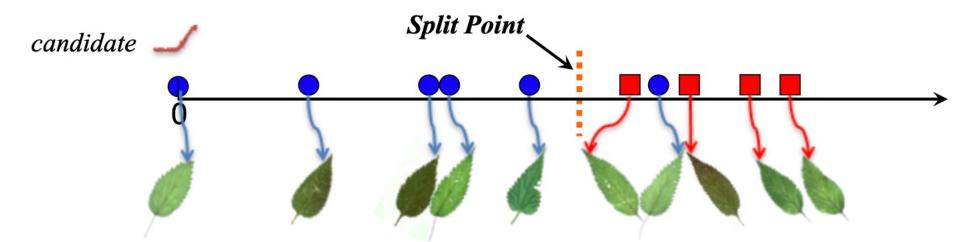
• The total number of candidate is

$$\sum_{l=MINLEN}^{MAXLEN} \sum_{T_i \in D} (|T_i| - l + 1)$$

- For each candidate you have to compute the distance between this candidate and each training sample (space inefficiency)
- For instance
  - 200 instances with length 275
  - 7,480,200 shapelet candidates

## Speedup

- Distance calculations form TSs to shapelet candidates is expensive.
- Reduce the time in two ways
- Distance Early Abandon
  - reduce the distance computation time between two TS
- Admissible Entropy Pruning
  - reduce the number of distance calculations

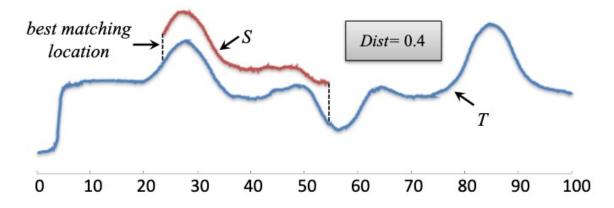


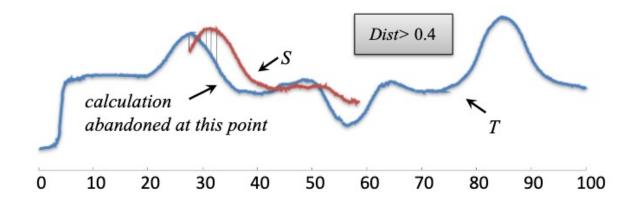
## Distance Early Abandon

We only need the minimum distance.

### Method

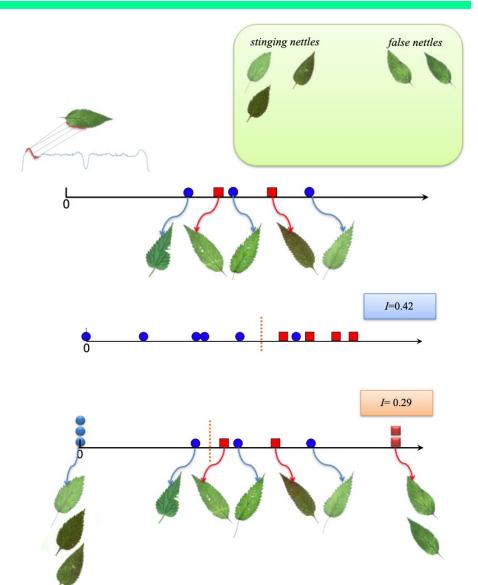
- Keep the best-so-far distance
- Abandon the calculation if the partial current distance is larger than best-so-far.
- We can avoid to compute the full distance for S if the partial one is greater than the best so far





## Admissible Entropy Pruning

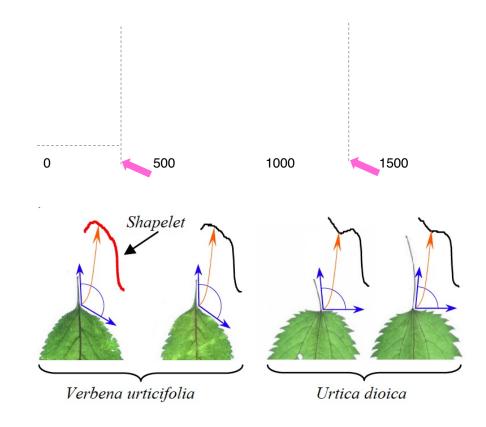
- We only need the best shapelet for each class
- For a candidate shapelet
  - We do not need to calculate the distance for each training sample
  - After calculating some training samples, the upper bound of information gain (corresponding to the optimistic scenario) < best candidate shapelet
  - Stop calculation
  - Try next candidate



# Motif/Shapelet Summary

 A motif is a repeated pattern/subsequence in a given TS.

 A shapelet is a pattern/subsequence which is maximally representative of a class with respect to a given dataset of TSs.



### References

- Matrix Profile I: All Pairs Similarity Joins for Time Series: A Unifying View that Includes Motifs, Discords and Shapelets. Chin-Chia Michael Yeh et al. 1997
- Time Series Shapelets: A New Primitive for Data Mining. Lexiang Ye and Eamonn Keogh. 2016.
- Josif Grabocka, Nicolas Schilling, Martin Wistuba, Lars Schmidt-Thieme (2014): Learning Time-Series Shapelets, in Proceedings of the 20th ACM SIGKDD Conference on Knowledge Discovery and Data Mining, KDD 2014

### Matrix Profile I: All Pairs Similarity Joins for Time Series A Unifying View that Includes Motifs, Discords and Shapelets

Chin-Chia Michael Yeh, Yan Zhu, Liudmila Ulanova, Nurjahan Begum, Yifei Ding Hoang Anh Dau, <sup>1</sup>Diego Furtado Silva, <sup>1</sup>Abdullah Mueen, and Eamonn Keogh University of California, Riverside, <sup>1</sup>Universidade de São Paulo, <sup>1</sup>University of New Mexico 115, Iulan001, nbegu001, yding007, hdun001)@ucc.edu, diegofulva@icmc.usp.br, mueen@uam.edu, eam

Advance—The elliptic indicative process for similarity jointy and the laws extensively studied for twa and a handful of substruction. However, surprisingly little progress has been studied as the extensive process of the surprise produce for two first that the desirable produces produce for two moderates that delense mental-loop groups produced for two first that the delense mental-loop groups produced for two first that the delense mental-loop groups produced for two first that the delense mental-loop groups produced for two first that the delense mental-loop groups produced for two first that the delense mental-loop groups produced for the delense for the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the delense of the delense of the groups produced for the delense of the del

by problem comes in several variants. The basic task is this: en a collection of data objects, retrieve the nearest neighbor each object. In the text domain the algorithm has

- tuning spatial access methods and/or hash function

- Our algorithm is embarrassingly parallelizable, both on multicore reconstructs and in distributed systems

Time Series Shapelets: A New Primitive for Data Mining

Dept. of Computer Science & Engineering University of California, Riverside, CA 92521 lexiangy@cs.ucr.edu

for most time series proteens. While this may be considered goo news, given the simplicity of implementing the nearest neighbor algorithm, there are some negative consequences of this. First, the nearest neighbor algorithm requires storing and searching the

### General Terms Algorithms, Experime

### 1. INTRODUCTION

In INTRODUCTION
While the last decode has seen a lunge interest in time series classification, to date the most accumte and rebust method is the simple nearest neighbor algorithm [412][31]. While the nearest neighbor algorithm has the advantages of simplicity and no requiring extensive parameter toning, it does have several important disorbantages. Chief among these are its space and time important disorbantages. Chief among these are its space and time.

Dept. of Computer Science & Engineering University of California, Riverside, CA 92521 eamonn@cs.ucr.edu

shows some examples of leaves from two classes, Urtica dioic (stinging nettles) and Verbusa articifolia. These two plants as



suppose we wish to build a classifier to distinute: what features should we use? Since the it



years [8]. However, here we find that using a nearest neigh to be due to the fact that the data is somewhat noisy (i.e. inse-bites, and different stem lengths), and this noise is enough t swamp the subtle differences in the shapes.