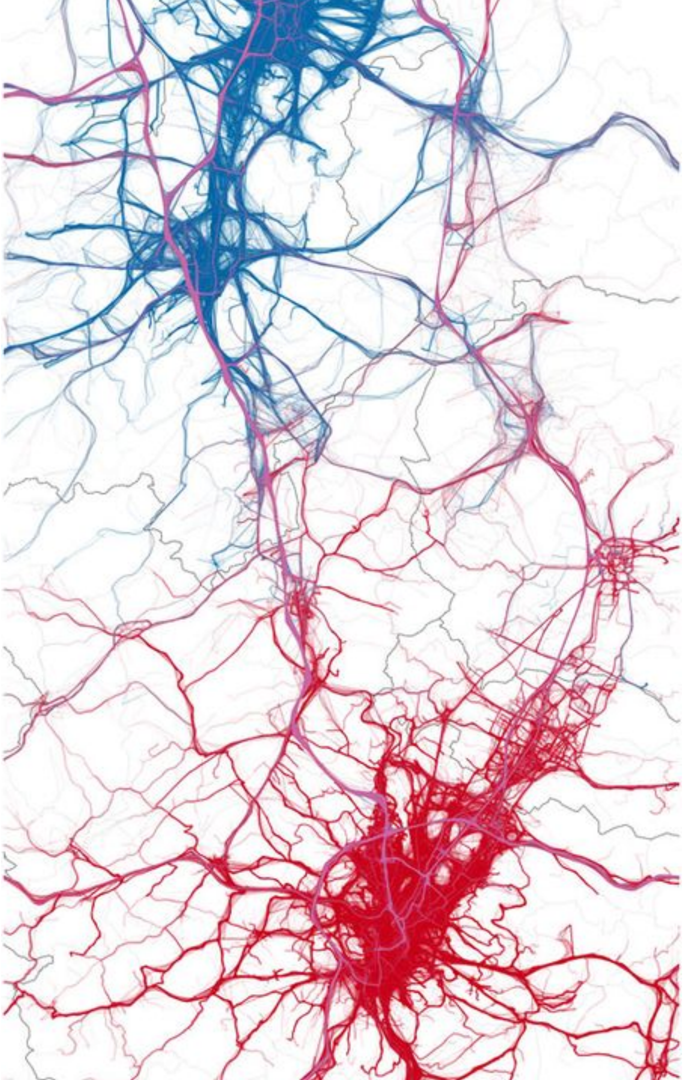


Consiglio Nazionale delle Ricerche

Collective mobility laws and models

Predictive vs Generative

- **predictive models**
  predict future trips/flows given past history of individuals
  - *machine learning, deep learning*

- **generative models**
  generate synthetic trajs or flows with realistic mobility patterns
  - *mechanistic modelling, machine learning, deep learning*

Individual vs Collective

- **individual models**
  generate/predict the trajectory of a single agent
  - *EPR and its variants*

- **collective models**
  generate/predict flows between locations
  - *Gravity, Radiation, Deep Gravity*

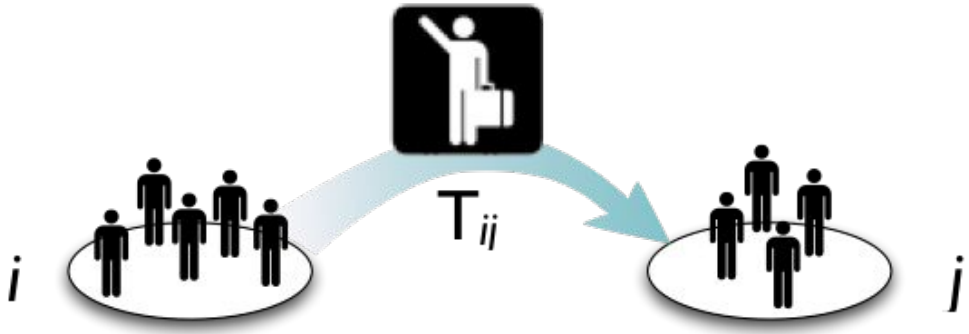
Collective models

generate mobility flows between origins and destinations

Spatial flows

are mathematically represented as an OD matrix T

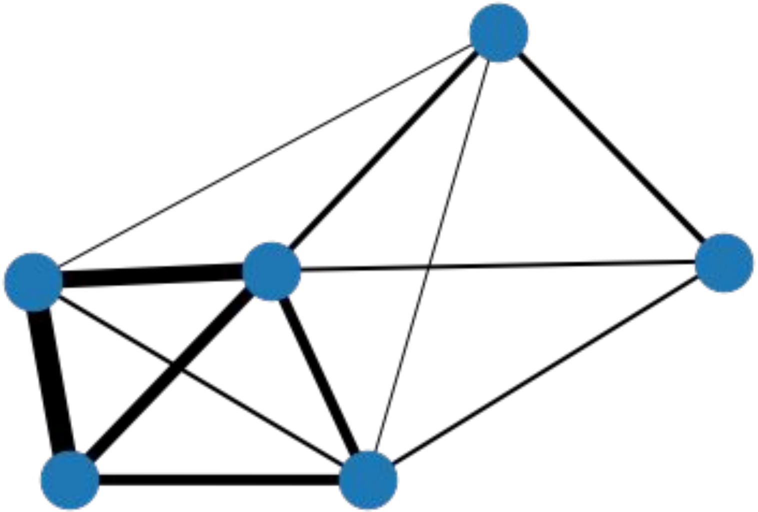
1. Define locations discretizing space (tessellation)
   e.g., counties, municipalities
2. T_{ij} is *the number of trips from i to j per unit time*.

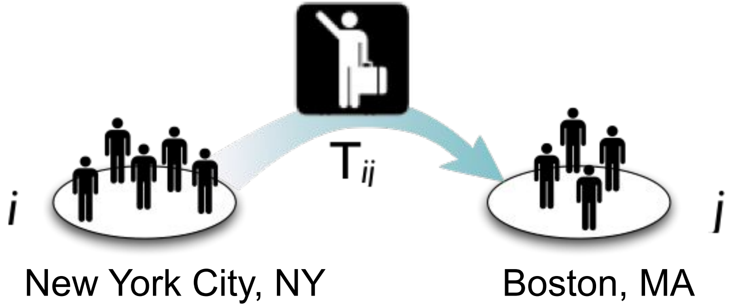


New York City, NY

Boston, MA

Spatial Flows



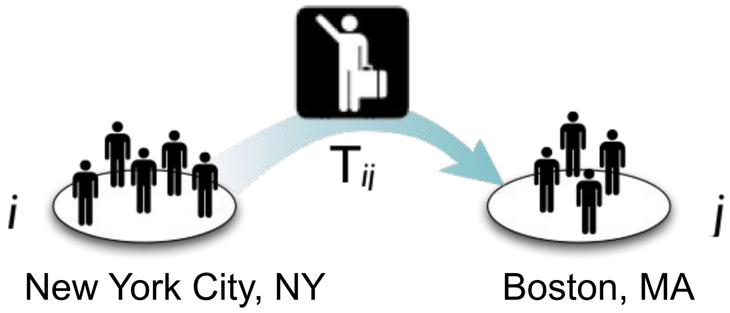


Spatial Flows

destination / origin

	a	b	c	d	e	f	
a	-	3	27	2	1	0	33
b	1	-	4	0	0	5	10
c	8	3	-	1	13	6	31
d	2	1	5	-	0	2	10
e	11	0	6	5	-	1	23
f	0	3	2	2	0	-	7
	22	10	44	10	14	14	114

(self-loops excluded)



total out-flow from i

$$\sum_j T_{ij} = O_i$$

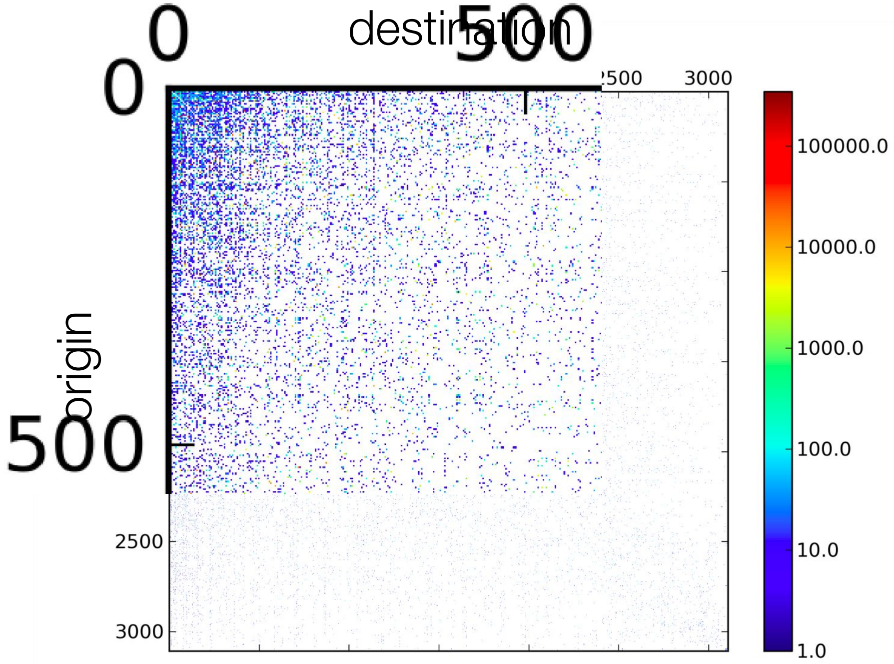
total in-flow to j

$$\sum_i T_{ij} = D_j$$

total flow

$$\sum_{ij} T_{ij} = N$$

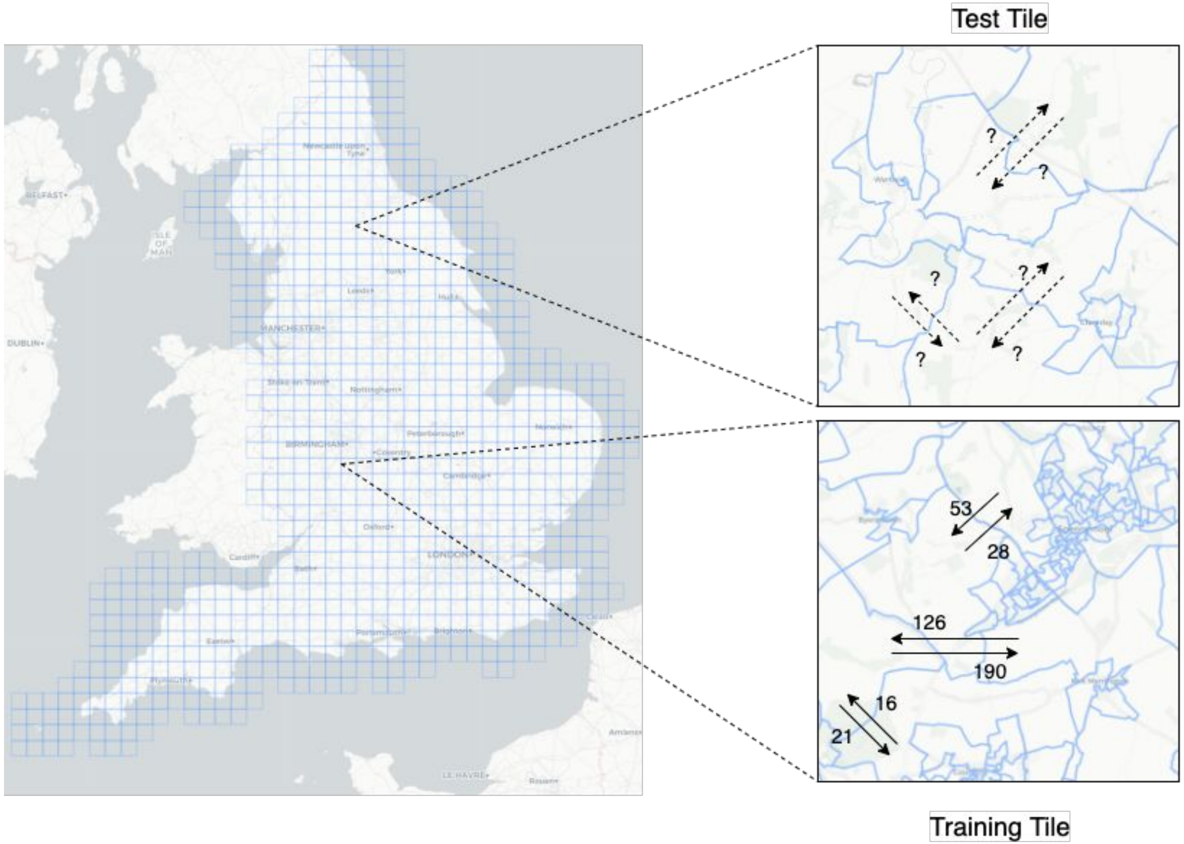
Spatial flows



US county to county commuting flows

Flow generation problem

generate realistic mobility flows among locations given their properties

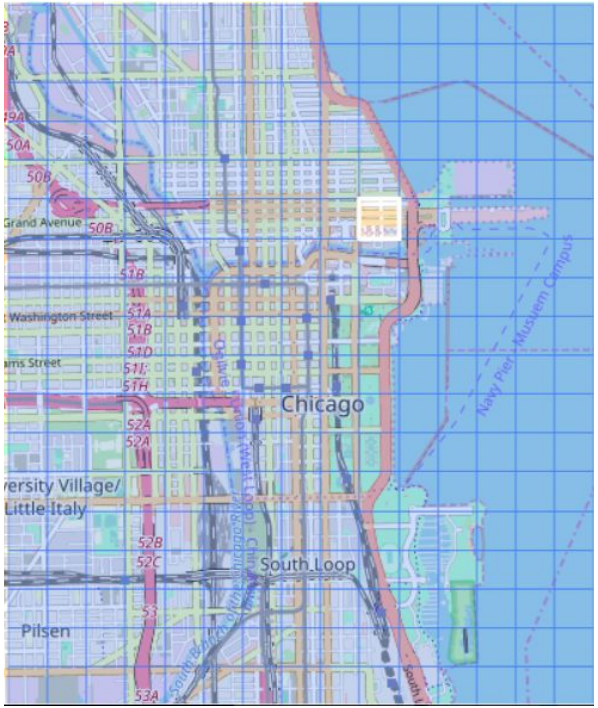


Flow generation problem

Interpret the problem as a classification task

classes = locations

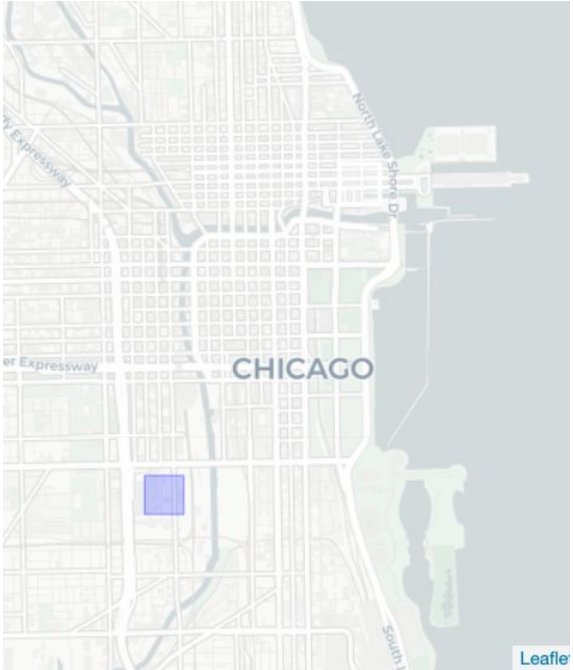
Probabilistic models



Interpret the problem as
a classification task

given a trip's origin location,
predict the destination

Probabilistic models



Goal: find the correct class (i.e., location of destination)

Each location has some probability to be the destination

How do we estimate these probabilities?

Probabilistic models

- assign a probability to each possible OD-matrix T

- fit model's parameters
    - maximizing the likelihood of observed T^*
    - minimizing the distance from observed T^*

Constrained models

- *globally* constrained (aka unconstrained) $$\sum_{ij} T_{ij} = N$$

singly

- *origin* constrained $$\sum_{j} T_{ij} = O_i \quad \forall i$$
- *destination* constrained $$\sum_{i} T_{ij} = D_j \quad \forall j$$

- *doubly* constrained $$\sum_{j} T_{ij} = O_i \qquad \sum_{i} T_{ij} = D_j$$

Properties of spatial flows

- Flows **decay** with distance

- Flows **grow** with population

- Flows **grow** with opportunities

Two main modelling approaches

1. Gravity (*G*) models
2. Intervening opportunities (*IO*) models

Similarities

Individual trips are independent. A trip's probability depends on:

- *weight*, an attribute of each individual location
  e.g., population, number of opportunities
- *distance*, a quantity relating a pair of locations

Differences

- different distance variables considered:
  - distance (G) vs # of intervening opportunities (IO)

Gravity model

Gravity model

Analogy with Newton's law of gravitation:

$$T_{ij} \propto \frac{P_i P_j}{r_{ij}} \longrightarrow T_{ij} = K m_i m_j f(r_{ij})$$

Gravity model

Analogy with Newton’s law of gravitation:

$$T_{ij} \propto \frac{P_i P_j}{r_{ij}} \longrightarrow T_{ij} = K m_i^{\alpha} m_j^{\beta} \boxed{f(r_{ij})}$$

$$f(r_{ij}) = r_{ij}^{\gamma}$$

power law

$$f(r_{ij}) = e^{\gamma r_{ij}}$$

exponential

$$f(r_{ij}) = \alpha r_{ij}^{\beta} e^{\gamma r_{ij}}$$

combination

the function’s optimal form may change according to:
the purpose of the trips, the spatial granularity, and the transportation mode

Constrained gravity models

The number of people originating from a location, or arriving to, are constrained to be a known quantity, and the gravity model is then used to estimate the destination:

proportionality constant

Singly constrained

$$T_{ij} = \boxed{K_i} O_i m_j f(r_{ij}) = O_i \frac{m_j f(r_{ij})}{\sum_k m_k f(r_{ik})} \qquad O_i = \sum_j T_{ij}$$

Globally constrained

$$T_{ij} = \boxed{K_i} O_i \boxed{L_j} D_j f(r_{ij}) \qquad D_j = \sum_i T_{ij}$$

$$K_i = \frac{1}{\sum_j L_j D_j f(r_{ij})} \quad L_j = \frac{1}{\sum_i K_i O_i f(r_{ij})}$$

Choosing the right gravity model

The use of singly-, doubly- or non-constrained models depends on the information available and on the objective:

- Aim: approximate the mobility flows and transport demand from indirect socio-economic variables
  → non-constrained models

- Out-going or in-going flows are empirically measured quantities, and the goal is to estimate the elements of the OD matrix
  → constrained models

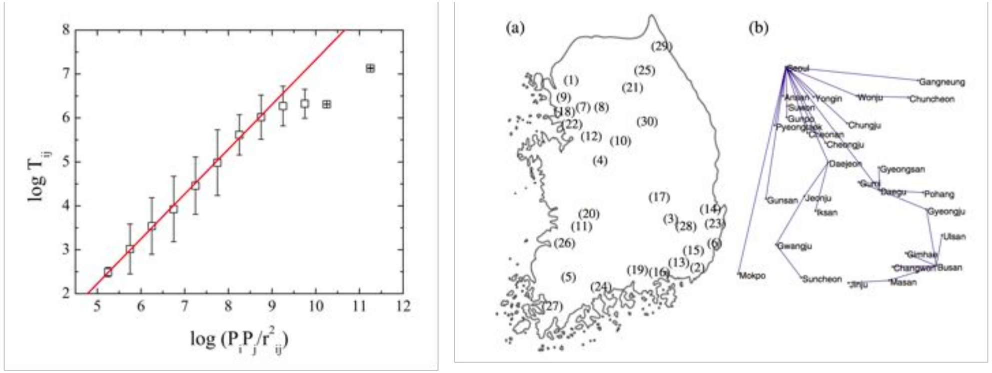
Fitting the gravity model

1. The set of independent variables (e.g., population size, gdp, distance) and the functions for these variables and the distance are established
   - power laws for populations
   - exponential or power laws for the distance dependence

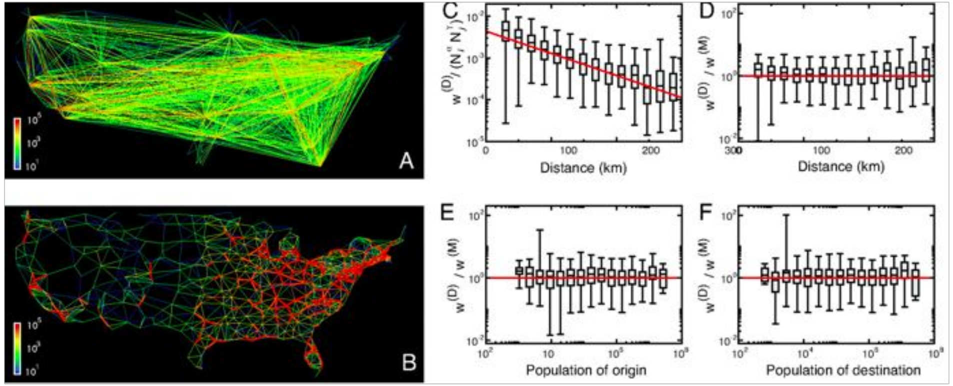
2. Parameter values are selected to maximize the fit between estimated and empirical flows:
   - best fit values determined using an optimization algorithm that minimizes some error function or maximizes the likelihood function of the observed data given the model's parameters

   - Generalized Linear Models (GLM) are usually applied to fit the parameters of globally and singly constrained gravity models

Gravity model: applications



Jung, W. S., Wang, F., & Stanley, H. E. (2008). Gravity model in the Korean highway. *EPL (Europhysics Letters)*, *81*(4), 48005.



Balcan, D., et al. "Multiscale mobility networks and the spatial spreading of infectious diseases." *PNAS* 106.51 (2009): 21484-21489.

Gravity model

PROs

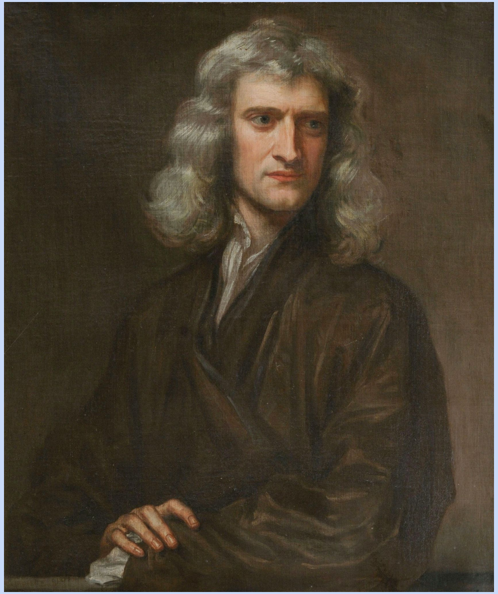
- parameters are easy to fit
- state-of-the-art performance
- versatility and wide applicability

CONs

- underfitting
- low generalisation power

INTERVALLO

Newton and the apple accident



Newton came up with his theory of universal gravitation as a result of an apple falling on his head.

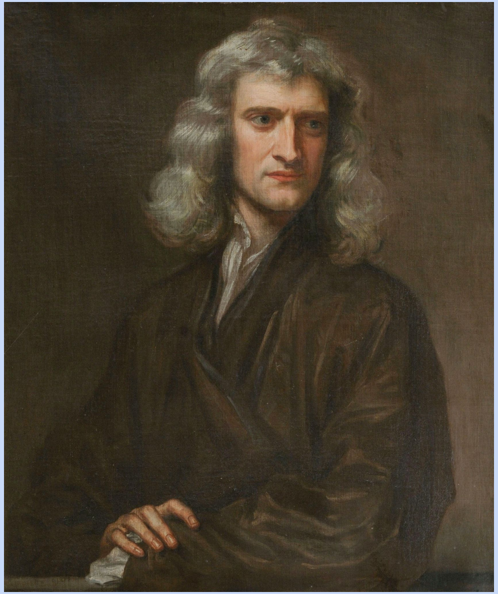
Is this story true?

YES!

Newton himself told the story many times and claimed that the incident had inspired him.

INTERVALLO

Newton and the apple accident



In his “Memoirs of Sir Isaac Newton’s Life” (1752), William Stukeley mentions a conversation in which Newton described pondering the nature of gravity while watching an apple fall:

“…we went into the garden, & drank thea under the shade of some apple trees; only he, & my self. amidst other discourse, he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. “why should that apple always descend perpendicularly to the ground,” thought he to himself; occasion’d by the fall of an apple…”

INTERVALLO

Where is Newton's apple tree?

Various trees are claimed to be "the" apple tree:

- The King's School in Grantham claims they purchased the original tree, uprooted it, and transported it to the headmaster's garden some years later;
- The National Trust, which holds the Woolsthorpe Manor (where Newton grew up) in trust, claims that the tree still resides in their garden.
- A descendant of the original tree can be seen growing outside the main gate of Trinity College, Cambridge, below the room Newton lived in when he studied there.

Intervening
opportunities

Intervening opportunities (IO)

“ The number of persons going a given distance is directly proportional to the number of opportunities at that distance and inversely proportional to the number of intervening opportunities ”

Stouffer, 1940

Stouffer, American Sociological Review 1940, Intervening opportunities: a theory relating mobility and distance

Intervening opportunities (IO)

Distance and mobility are not directly related:

- what plays the key role in determining migration is the **number of intervening opportunities** between the origin and the destination

- Stouffer does not provide a precise definition for “opportunities”, leaving it to be defined depending on the social phenomena under investigation

Stouffer, American Sociological Review 1940, Intervening opportunities: a theory relating mobility and distance

Intervening opportunities (IO)

The decision to make a trip is explicitly related to the relative accessibility of opportunities for satisfying the objective of the trip:

- an opportunity is a destination that a trip-maker considers as a possible termination point for their journey

- an intervening opportunity is a location that is closer to the trip maker than the final destination but is rejected by the trip maker

Stouffer, American Sociological Review 1940, Intervening opportunities: a theory relating mobility and distance

Intervening opportunities (IO)

“The probability that a trip ends in a given location is equal to the probability that this location offers an acceptable opportunity times the probability that an acceptable opportunity in another location closer to the origin of the trips has not been chosen.”

Schneider, 1959

M. Schneider, Gravity models and trip distribution theory, Papers of the regional science association 5 (1959) 51–58.

Intervening opportunities (IO)

cumulative number of opportunities up to the j-th location ranked by travel cost from origin location

$$T_{ij} = O_i \frac{e^{-L V_{i,j-1}} - e^{-L \boxed{V_{i,j}}}}{\underbrace{\boxed{1 - e^{-L V_{i,n}}}}_{\text{normalization factor}}}$$

- Usually, the population or the total number of arrivals are assumed to be proportional to the number of “real opportunities” in a location
- L is the constant probability of accepting an opportunity destination
  - As in the case of the gravity model, the value of L is adjusted in order to obtain simulated flows as close as possible to observed data

Radiation model

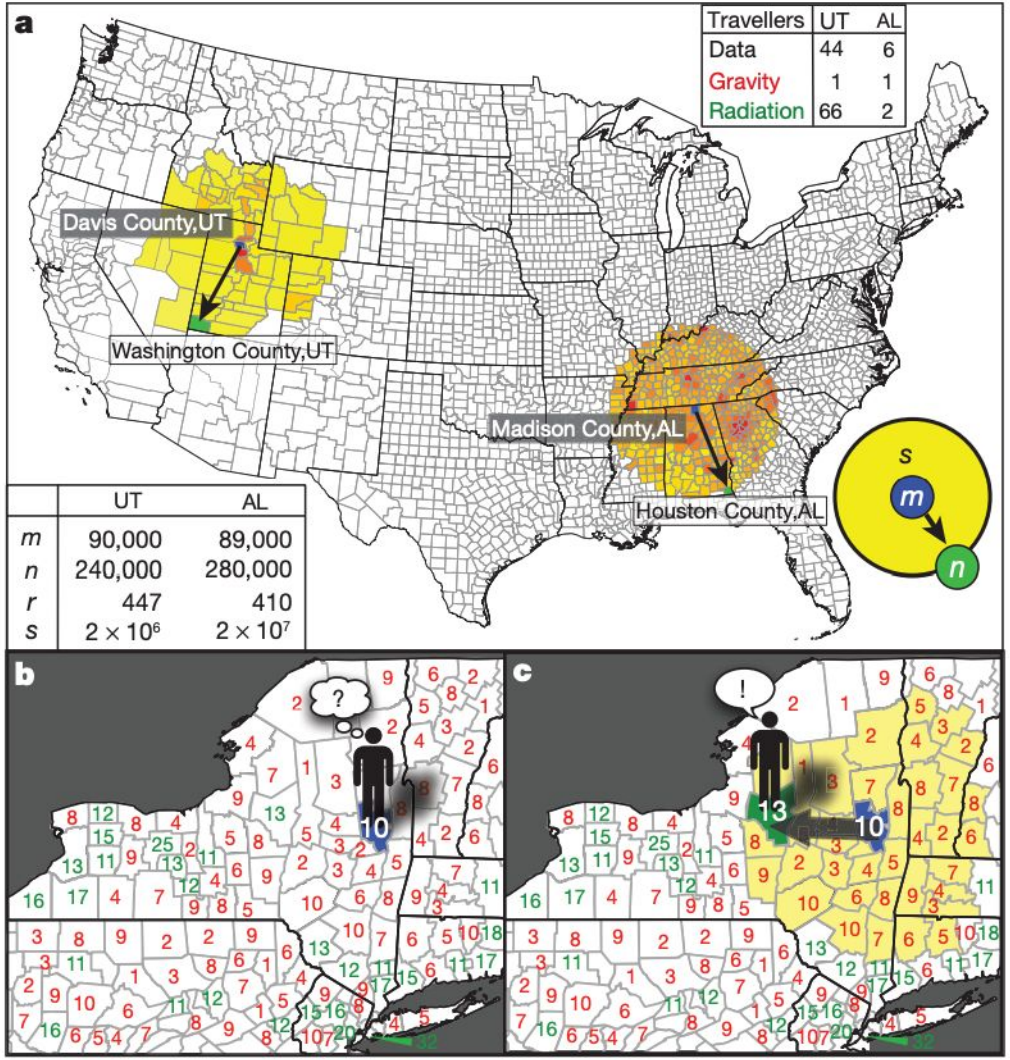
The radiation model elaborates on the IO hypothesis and assumes that the choice of a traveler's destination consists of these steps:

1. each opportunity in every location is assigned a **fitness** z chosen from distribution $p(z)$, (quality of the opportunity for the traveler)

2. the traveler ranks all opportunities according to their distance from the origin location

3. the traveler chooses the closest opportunity with a fitness higher than the traveler's fitness threshold (randomly extracted from $p(z)$)

Simini et al., Nature 2012 A universal model for mobility and migration patterns



- Each opportunity has a “value”, extracted from some distribution.
- Each individual has expectations, extracted from the same distribution.
- Principle of least effort: each individual chooses the closest opportunity that meets their expectations

Radiation model

Parameter-free: the model depends only on the populations

opportunities at the origin

opportunities at the destination

$$T_{ij} = O_i \boxed{\frac{1}{1-\frac{m_i}{M}}} \frac{\boxed{m_i}\boxed{m_j}}{(m_i+\boxed{s_{ij}})(m_i+m_j+s_{ij})}$$

normalization factor
(so that the probability that a trip originating in the region ends in this location is 1)

opportunities in a circle of radius r_{ij} centered in the origin location i (excluding origin and destination)

Intervening Opportunities

PROs

- parameter-free (Radiation and PWO)
- performance comparable to Gravity models

CONs

- underfitting
- overdispersion

Other collective models

Several other others have been proposed so far; they are typically variants of G, IO or Radiation:

- Rank-distance model $$p_{ij} \propto \frac{1}{(m_i + s_{ij})^\alpha}$$

- Population-weighted opportunities (PWO)
  - considers the opportunities centered at the destination

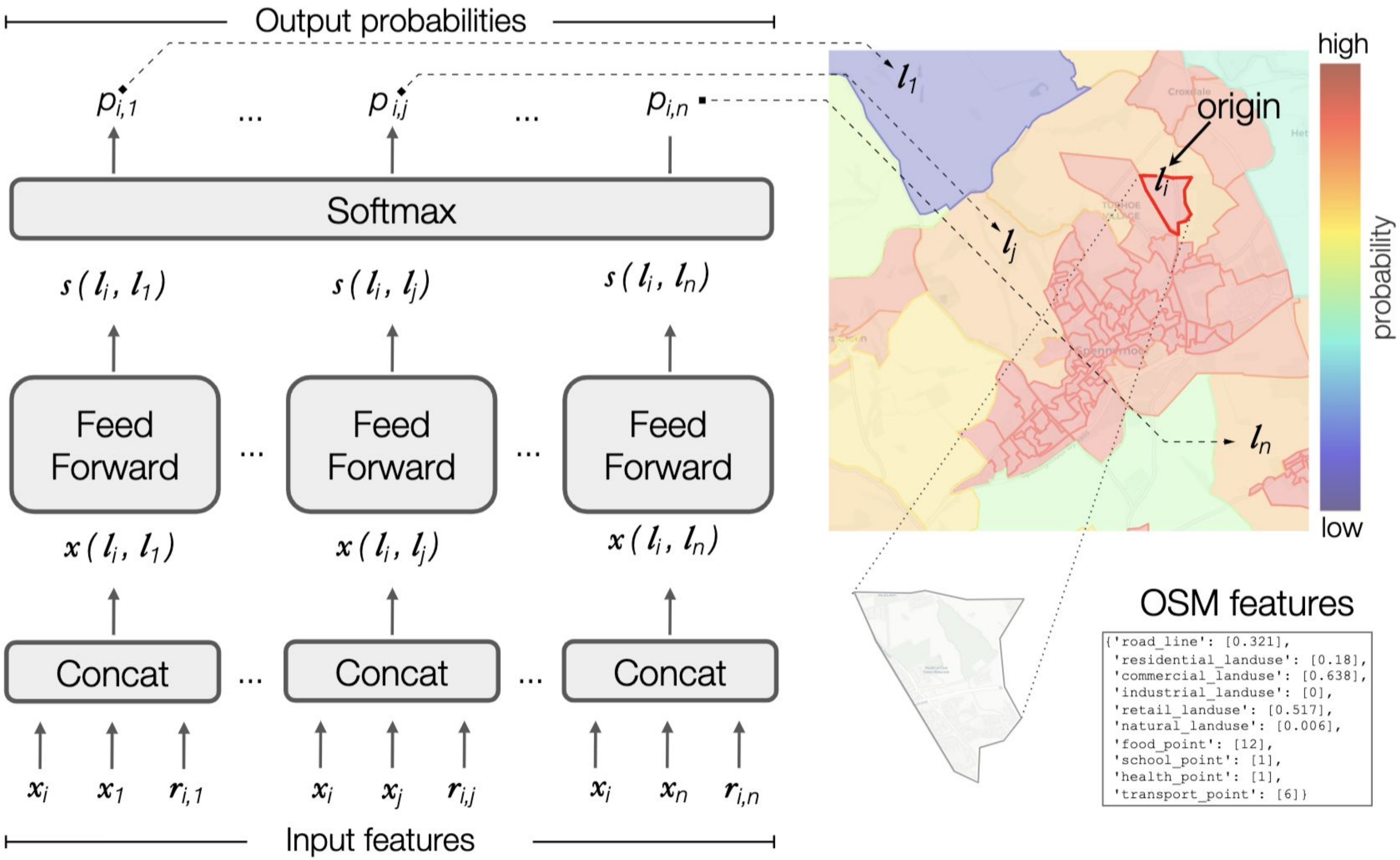
$$p_{ij} \propto m_j \left(\frac{1}{m_i + m_j + s_{ji}} - \frac{1}{M} \right)$$

Deep Gravity

1. Capture non-linear relationships using deep neural networks

2. Characterize locations better using alternative data sources (e.g., POIs from OpenStreetMap)

3. Using explainable AI techniques to gain a deeper understanding of the patterns underlying mobility flows

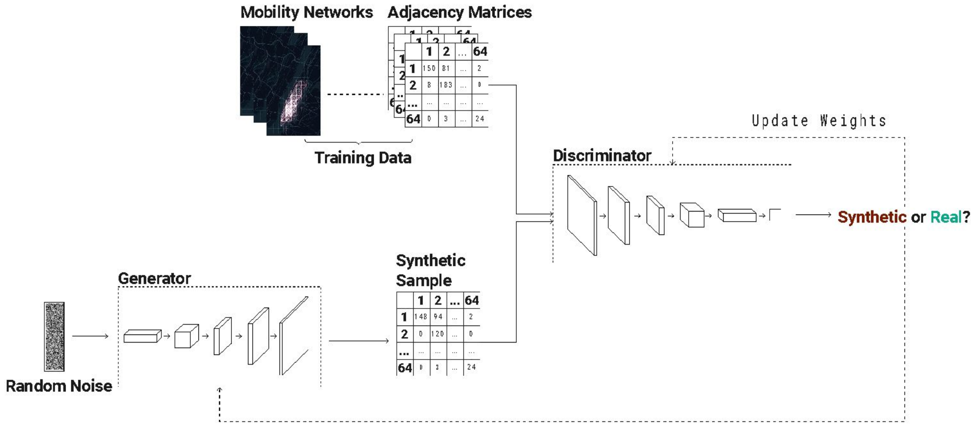

Output probabilities
$p_{i,1}$ ... $p_{i,j}$ ... $p_{i,n}$
Softmax
$s(l_i, l_1)$
$s(l_i, l_j)$
$s(l_i, l_n)$
Feed Forward
Feed Forward
Feed Forward
$x(l_i, l_1)$
$x(l_i, l_j)$
$x(l_i, l_n)$
Concat
Concat
Concat
x_i x_1 $r_{i,1}$
x_i x_j $r_{i,j}$
x_i x_n $r_{i,n}$
Input features
l_1
origin
l_i
l_j
l_n
high
probability
low
OSM features
{'road_line': [0.321],
'residential_landuse': [0.18],
'commercial_landuse': [0.638],
'industrial_landuse': [0],
'retail_landuse': [0.517],
'natural_landuse': [0.006],
'food_point': [12],
'school_point': [1],
'health_point': [1],
'transport_point': [6]}

Input Data

Category	# feat.	Description
Land use areas	3	total area (in km^2) for each possible land use class
Road network	3	total length (in km) for each type of road network
Transportation	2	# POIs, building related to each possible transport facility
Food	2	# POIs, building related to each possible food facility
Health	2	# POIs, building related to each possible health facility
Education	2	# POIs, building related to each possible education facility
Retail	2	# POIs, building related to each possible retail facility
Distance	1	Distance between two locations

Other flow generation models

MoGAN:
generating flows with GANs



LLMs-based flow generation

GPT-4

Validation of collective models

Common metrics to compare OD matrices

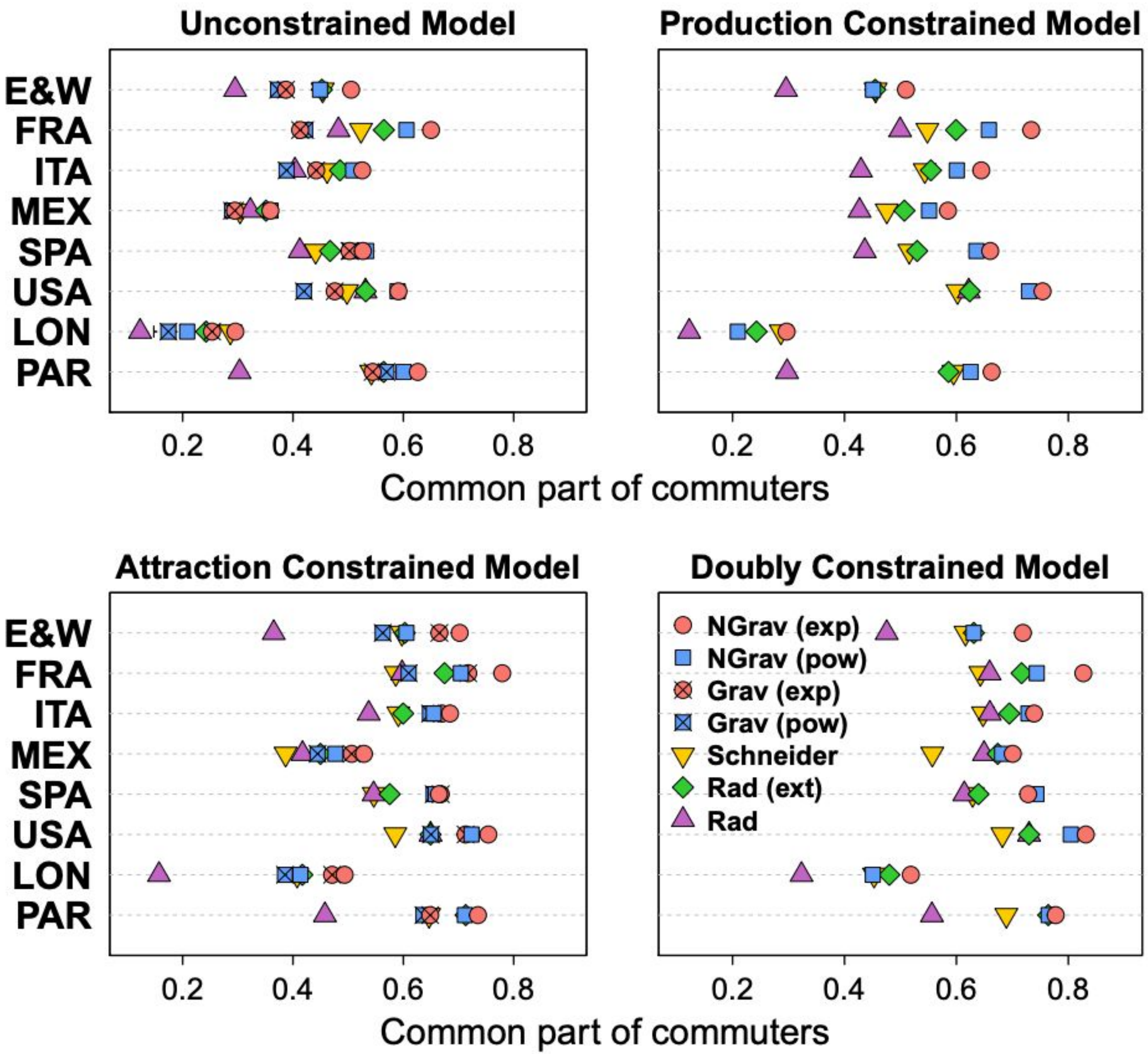
- Sorensen-Dice similarity
  (Common part of commuters)

$$\frac{\sum_{ij} \min(T_{ij}^{e}, T_{ij}^{m})}{\sum_{ij} T_{ij}^{e}}$$

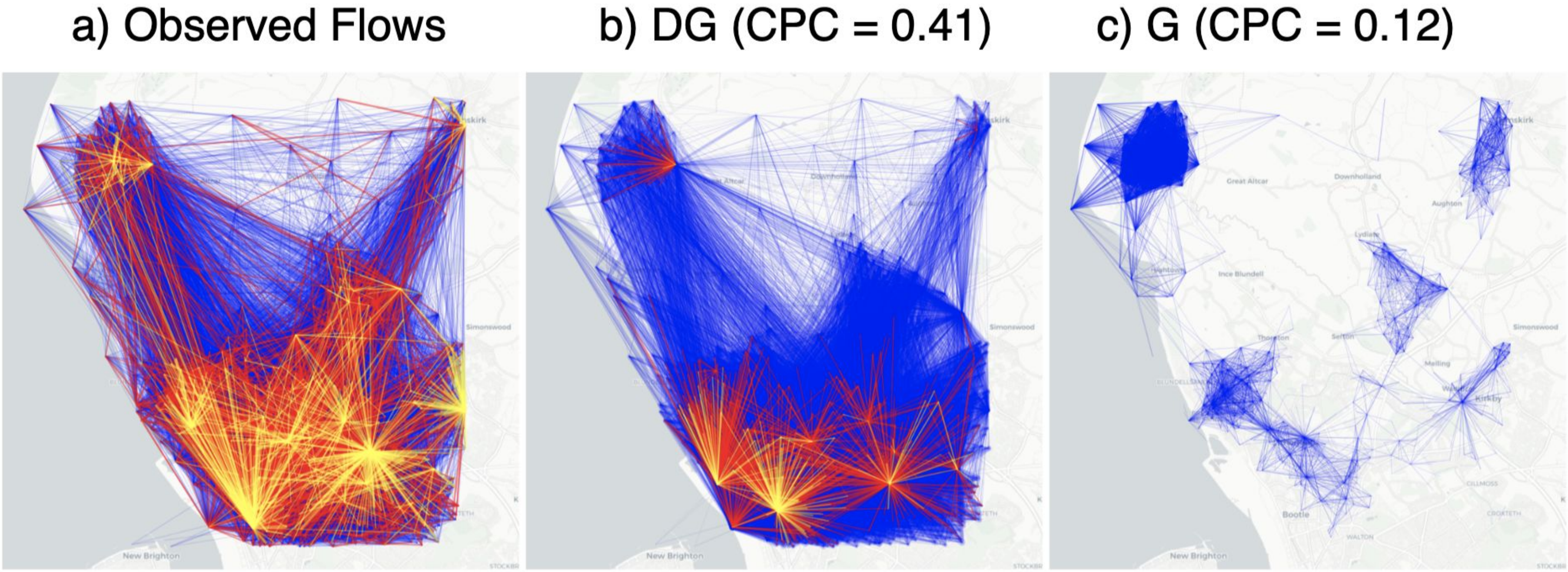
- Root Mean Squared Error

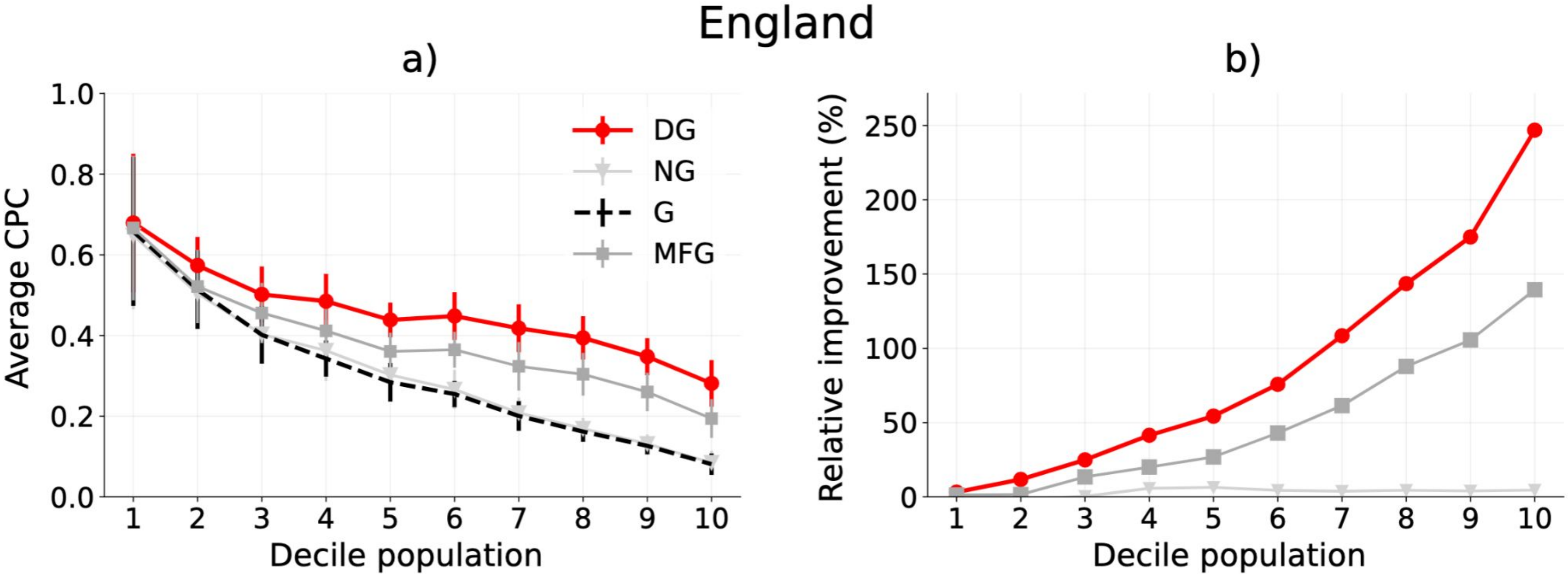
$$\sqrt{\frac{\sum_{ij}(T_{ij}^{e} - T_{ij}^{m})^2}{n^2}}$$

- More (cosine similarity, correlation, …)



Unconstrained Model
Production Constrained Model
Attraction Constrained Model
Doubly Constrained Model
E&W
FRA
ITA
MEX
SPA
USA
LON
PAR
0.2
0.4
0.6
0.8
Common part of commuters
NGrav (exp)
NGrav (pow)
Grav (exp)
Grav (pow)
Schneider
Rad (ext)
Rad


a) Observed Flows
b) DG (CPC = 0.41)
c) G (CPC = 0.12)
Simonswood
New Brighton
Great Altcar
Downholland
Aughton
Ince Blundell
Bootle


England
a)
b)
Average CPC
1.0
0.8
0.6
0.4
0.2
0.0
DG
NG
G
MFG
1
2
3
4
5
6
7
8
9
10
Decile population
Relative improvement (%)
250
200
150
100
50
0
1
2
3
4
5
6
7
8
9
10
Decile population

to study for the exam

References

- [paper] Human Mobility: Models and Applications, Barbosa et al., Physics Report, 2018, Section 4.2

- [paper] Systematic comparison of trip distribution laws and models, Lenormand et al., Journal of Transport Geography, 2016

- [paper] A Deep Gravity model for mobility flows generation, Simini et al., Nature Communications, 2021

Homework

Download the flows for at least two different US States from this repository, create and plot a FlowDataFrame. Then:

- split the FlowDataFrame into a training set and a test set;
- train the Gravity and Radiation models on the training set
- test the models' goodness on the test set (qualitative and quantitative evaluation). Use population as location relevance.
- Compare the two models with appropriate plots and/or tables.
- Repeat using the number of Education facilities in each location instead of the popultion (i.e., total count of POIs and buildings related to all education facilities, e.g., school, college, kindergarten, etc.).

Submit a well-commented notebook.