02

Fundamental Concepts
Geographic Information Systems (GIS)

- A computer system for capturing, storing, querying, analyzing, and displaying geospatial data.

- Geospatial data describe both the locations and characteristics of spatial features, e.g.:
  - to describe a road or a restaurant, we refer to its location (i.e., where it is) and its characteristics;
  - to describe a trajectory, we refer to the sequence of locations.
Fundamental concepts

- **Geographic Coordinate Systems**
  reference system for locating points on the Earth’s surface

- **Vector data model**
  representation of spatial features in GIS

- **Trajectory**
  sequence of points that describe an individual’s movements

- **Spatial tessellation**
  division of the space into non-overlapping tiles

- **Flows**
  movements of groups of people between places
Geographic Coordinate Systems
reference system for locating points on Earth’s surface
Geographic Coordinate System (GCS)

Reference system for locating points on the Earth’s surface, based on two angles:

- **longitude** (Long)
  - angle E/W from the Prime Meridian (PM)

- **latitude** (Lat)
  - angle N/S of the equatorial plane
Latitude (North/South)

90°N

45°N

0°

45°S

90°S

Latitude varies from 0° at the equator to 90° North and South at the poles

Longitude (West/East)

Equator

W

E

Longitude varies from 0° at Greenwich to 180° East and West
Geographic Coordinate System (GCS)

- **Meridians**: lines of equal Long
  - PM passes through **Greenwich, UK** (0°)
  - Long $\in [0^\circ, 180^\circ]$ E/W of PM
Geographic Coordinate System (GCS)

- **Parallels**: lines of equal Lat
  - $\text{Lat} \in [0^\circ, 90^\circ]$ N/S of the equator (0°)
Geographic Coordinate System (GCS)

- **Meridians**: lines of equal Long
  - PM passes through Greenwich, UK (0°)
  - Long ∈ [0°, 180°] E/W of PM

- **Parallels**: lines of equal Lat
  - Lat ∈ [0°, 90°] N/S of the equator (0°)
Geographic Coordinate System (GCS)

In a *plane coordinates* perspective:

- Long values are $x$ values
  - Positive in eastern hemisphere, negative in western
  - $\in [-180^\circ, 180^\circ]$ 

- Lat values are $y$ values
  - Positive if north of equator, negative otherwise
  - $\in [-90^\circ, 90^\circ]$
Geographic Coordinate System (GCS)

Long and Lat may be expressed in:

- **decimal degrees** (DD)
- **degrees-minutes-seconds** (DMS)
- **radians** (rad)

Conversion DMS to DD:

- 1 degree = 60 minutes; 1 minute = 60 seconds
- 45°52'30" (Lat) is 45.875° (45 + 52/60 + 30/3600)
Geographic Coordinate System (GCS)

Long and Lat may be expressed in:

- **decimal degrees (DD)**
- **degrees-minutes-seconds (DMS)**
- **radians (rad)**

Conversion DMS to rad:

- \( \text{rad} = \text{DD} \times \frac{\pi}{180} \)
- one radian equals 57.2958°
- one degree equals 0.01745 rad
Geodetic Datum

Mathematical model of the Earth, serving as the reference for calculating the geographic coordinates

1. Long & Lat of an initial point (origin)
2. reference ellipsoid model, approximating Earth’s shape

Several local and global geodetic datums have been proposed

- A point on Earth can have different coordinates depending on the datum used
World Geodetic System 1984 (WGS84 or EPSG:4326)

- Used by the U.S. Department of Defense as global reference system for supporting positioning and navigation
- Datum for GPS readings:
  - satellites send their positions in WGS84 coordinates,
  - calculations in GPS receivers are based on WGS84
- Meridian of zero Long is the IERS Reference Meridian (close to Greenwich meridian) at the Lat of the Royal Observatory
- Earth surface is an **oblate spheroid** with equatorial radius:
  - $a = 6378137$ m at the equator
  - flattening $f = 1/298.257223563$. 
Distance on the Earth surface

Straight lines are replaced by geodesics (great circles):

- Through two points (not antipodal) there is a unique great circle
- The two points separate the great circle into two arcs:
  - the shortest (red) is the great-circle distance
- The Earth is nearly spherical: great-circle distance are correct to within about 0.5%
Distance on the Earth surface

Let P and Q be two points, the central angle between them is:

\[ \Delta \sigma = \frac{d}{r} \]

spherical distance

sphere’s radius
We can use the **haversine formula** to compute the great-circle distance between any two points given their Long and Lat:

\[
hav(\Delta \sigma) = hav \left( \frac{d}{r} \right)\]
Haversine formula

Apply the arcsine (inverse sine) function:

\[ d = 2r \arcsin \left( \sqrt{\text{hav}(\Delta \sigma)} \right) \]

\[ d = 2r \arcsin \left( \sqrt{\sin^2 \left( \frac{\Delta \text{Lat}}{2} \right) + \cos \text{Lat}_P \cdot \cos \text{Lat}_Q \cdot \sin^2 \left( \frac{\Delta \text{Long}}{2} \right)} \right) \]
Haversine formula

- An *approximation*, because the Earth is not a perfect sphere:
  - Earth radius varies from 6356.752km at the poles to 6378.137 km at the equator
  - the radius of curvature of a north-south line on the Earth's surface is 1% greater at the poles (≈6399.594 km) than at the equator (≈6335.439 km)

- The haversine formula and law of cosines cannot be guaranteed correct to better than 0.5%
What if...Earth would be flat?
just an Euclidean distance between points would be ok

\[ D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]
Did you know that...

Globo crucigero

- A golden sphere with a cross on the top
- It represents the supremacy of Christ over the world and, if held in the hand of a ruler, the divine legitimacy of power

It is not true that ancient and medieval people believed in flat Earth!
It is not true that ancient and medieval people believed in flat Earth!

Globo crucigero

- A golden sphere with a cross on the top
- It represents the supremacy of Christ over the world and, if held in the hand of a ruler, the divine legitimacy of power
The Flat Earth theory is recent!

- **XIX century:** the belief of flat Earth starts diffusing in England and the US (Rowbotham, Blount, Carpenter)

- **England, 1956:** Samuel Shenton found the Flat Earth Society
The Flat Earth theory is recent!

- XIX century: the belief of flat Earth starts diffusing in England and the US (Rowbotham, Blount, Carpenter)
- England, 1956: Samuel Shenton found the Flat Earth Society
- 1956: The Flat Earth Society has members all around the globe.
Vector data model
representation of spatial features in GIS
Vector data model

It uses discrete objects to represent spatial features:

1. representing points, lines, and polygons on an empty space

2. structuring the properties and spatial relationships of these geometric objects

3. coding and storing vector data in digital data files
Vector types

POINTS

LINES

POLYGONS
Vector types

- **Point**: zero dimension
  - properties: *location* (xy coords)
Vector types

- **Point**: zero dimension
  - properties: *location* (xy coords)

- **Line**: one-dimensional
  - properties: *location* and *length*
  - has two end *Points*
  - straight-line or curve
Vector types

- **Point**: zero dimension
  - properties: location (xy coords)

- **Line**: one-dimensional
  - properties: location and length
  - has two end Points
  - straight-line or curve

- **Polygon**: two-dimensional
  - properties: location, area, perimeter
  - made of connected closed lines
Point (node, vertex)
INDIVIDUAL X,Y LOCATIONS
E.g., label, manhole, tower locations

Line (chain)
2 OR MORE POINTS THAT ARE CONNECTED
E.g., paths, roads, streams

Polygon (area segment)
3 OR MORE VERTICES THAT ARE CONNECTED AND CLOSED
E.g., land/water boundaries, buildings

- **Google Maps**
  - Point, Linestring, Linering, Polygon

- **GeoJSON**
  - Point, LineString, Polygon

- **Shapely**
  - Point, LineString, Polygon
Georelational Data Model

How do we represent geometric objects in a computer?

- Georelational data model: stores geometries and attributes separately
  - Topological: coverage
  - Non-topological: shapefile
Coverage: Point

Point list

<table>
<thead>
<tr>
<th>ID</th>
<th>x, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>2</td>
<td>(6, 2)</td>
</tr>
<tr>
<td>3</td>
<td>(4, 4)</td>
</tr>
<tr>
<td>4</td>
<td>(2, 9)</td>
</tr>
</tbody>
</table>
### Point list

<table>
<thead>
<tr>
<th>ID</th>
<th>x, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>(0, 9)</td>
</tr>
<tr>
<td>12</td>
<td>(2, 9)</td>
</tr>
<tr>
<td>13</td>
<td>(8, 9)</td>
</tr>
<tr>
<td>14</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>15</td>
<td>(4, 2)</td>
</tr>
<tr>
<td>16</td>
<td>(4, 0)</td>
</tr>
</tbody>
</table>

### Arc-node list

<table>
<thead>
<tr>
<th>Arc</th>
<th>F-node</th>
<th>T-node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>16</td>
</tr>
</tbody>
</table>

**Coverage: Line**
### Arc-coordinate list

<table>
<thead>
<tr>
<th>Arc</th>
<th>coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0,9) (2, 9)</td>
</tr>
<tr>
<td>2</td>
<td>(2, 9) (8, 9)</td>
</tr>
<tr>
<td>3</td>
<td>(2, 9) (2,6) (4, 4) (4, 2)</td>
</tr>
<tr>
<td>4</td>
<td>(8, 9) (8,7) (7, 5) (6, 2) (4, 2)</td>
</tr>
<tr>
<td>5</td>
<td>(4, 2) (1, 2)</td>
</tr>
<tr>
<td>6</td>
<td>(4, 2) (4, 0)</td>
</tr>
</tbody>
</table>

**Coverage: Line**
<table>
<thead>
<tr>
<th>ID</th>
<th>x, y</th>
<th>Arc</th>
<th>Arc coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>(4, 9)</td>
<td>1</td>
<td>(1, 3) (1, 9) (4, 9)</td>
</tr>
<tr>
<td>12</td>
<td>(9, 6)</td>
<td>2</td>
<td>(4, 9) (9, 9) (9, 6)</td>
</tr>
<tr>
<td>13</td>
<td>(1, 3)</td>
<td>3</td>
<td>(9, 6) (9, 1) (1, 1) (1, 3)</td>
</tr>
<tr>
<td>14</td>
<td>(5, 3)</td>
<td>4</td>
<td>(4, 9) (4, 7) (5, 5) (5, 3)</td>
</tr>
<tr>
<td>15</td>
<td>(5, 7)</td>
<td>5</td>
<td>(9, 6) (7, 3) (5, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>(5, 1) (1, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>(5, 7) (6, 8) (7, 7) (7, 6) (5, 6) (5, 7)</td>
</tr>
</tbody>
</table>

Coverage: Polygon
## Polygon-arc list

<table>
<thead>
<tr>
<th>Polygon</th>
<th>arc</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>1, 4, 6</td>
</tr>
<tr>
<td>102</td>
<td>4, 2, 5, 0, 7</td>
</tr>
<tr>
<td>103</td>
<td>6, 5, 3</td>
</tr>
<tr>
<td>104</td>
<td>7</td>
</tr>
</tbody>
</table>

**Coverage: Polygon**
Trajectory
sequence of points that describe people’s movements
What is a trajectory?
Let \( u \) be an individual, a trajectory

\[
T_u = \langle p_1, p_2, \ldots, p_{nu} \rangle
\]

is a time-ordered sequence composed by the spatio-temporal points \( u \) visited.

A spatio-temporal point is a pair

\[
\mathbf{p} = (t, \mathbf{l})
\]

where:

- \( t \) is the time
- \( \mathbf{l} = (x, y) \) is the point visited
  - \( x \) and \( y \) are spatial coords
Spatial tessellation
division of the space into non-overlapping tiles
Definition **Spatial tessellation**
Given an area $A$, a tessellation is a set of geographical polygons with the following properties:
Given an area $A$, a tessellation is a set of geographical polygons with the following properties:

1. it contains a finite number of polygons, called tiles

$$ \mathcal{G} = \{g_i : i = 1, \ldots, n\} $$
Given an area $A$, a tessellation is a set of geographical polygons with the following properties:

1. it contains a finite number of polygons, called tiles
   
   
   $$\mathcal{G} = \{g_i : i = 1, \ldots, n\}$$

2. the tiles are non-overlapping
   
   
   $$g_i \cap g_j = \emptyset, \forall i \neq j$$
Given an area $A$, a tessellación is a set of geographical polygons with the following properties:

1. it contains a finite number of polygons, called tiles
   \[ \mathcal{G} = \{ g_i : i = 1, \ldots, n \} \]
2. the tiles are non-overlapping
   \[ g_i \cap g_j = \emptyset, \forall i \neq j \]
3. the union of all tiles completely covers the tessellación
   \[ \bigcup_{i=1}^{n} g_i = A \]
A tessellation may be:

- **Regular**
  - equilateral triangular, squared, quadrilateral, hexagonal

- **Irregular**
  - buildings, census cells, administrative units

- A spatial join can be used to associate a point with the tile that contains it

- Since the tessellation has no overlapping tiles and no gaps, each point is assigned only to one tile
● Uber H3 recursively creates increasingly higher precision hexagon grids

● Each subsequent resolution is created splitting each cell into seven children recursively

● 16 grid resolutions:
  ○ resolution 0: 122 base cells
  ○ resolution 15: cells with an area of less than $1 \text{ m}^2$
- S2 Geometry decomposes the unit sphere into a hierarchy of cells (quadrilateral bounded by four geodesics)
- The top level projects the six faces of a cube into the unit sphere
- Lower levels subdivisions each cell into four sub-cells recursively
- Each cell in the hierarchy has a level, defined as the number of times the cell has been subdivided (starting with a face cell).
- Cells’ levels range from 0 to 30. The smallest cells at level 30 are called leaf cells; there are $6 \times 4^{30}$ cells in total, each about 1cm across on the Earth’s surface
VORONOI TESSELLATIONS

- A partition of a plane into regions close to each of a given set of objects.
- For object there is a corresponding region, called a Voronoi cell, consisting of all points of the plane closer to that seed than to any other.
Tessellations in natural systems
Tessellations in natural systems
Tessellations in natural systems
Tessellations in artificial systems
References

[article] Voronoi Tessellations and Scutoids Are Everywhere, Scientific American, 2019
Flows

movements of groups of people between places
Given a tessellation, the flow:

\[ y(g_i, g_j) \]

represents the number of people (or in general objects) moving between \( g_i \) and \( g_j \).

**Trajectory vs Flows:**

- A trajectory corresponds to a single individual
- A flow corresponds to the total amount of people that move between two points
- Flows can be obtained from a set of trajectories
- Trajectories cannot be obtained from flows
Material

  - Chapter 3: Vector Data model

- [lesson] *Automating GIS-processes*, Department of Geosciences and Geography, University of Helsinki, Finland
  - Lesson 1, *Shapely and geometric objects*

  - Chapter 4, Section 4.2: *Vector Data Models*

- [article] *Analyze Geospatial Data in Python: GeoPandas and Shapely*, learndatasci.com
Material

  - Chapter 2 (Section 2.1)

  - Section 2.1, Appendix A

- [paper] *scikit-mobility: a Python library for the Analysis, Generation, and Risk Assessment of Mobility Data*, Pappalardo et al., Journal of Statistical Software
  - Sections 1, 2
Material

- [video] Intro to coordinate systems and UTM projection, Middlebury Remote Sensing, 2016
Homeworks
to be delivered by Friday, 23rd/29th 2022
Homework 2.1

Compute the distance from your home to the five largest capitals in the EU, using both the Earth distance and the Euclidean distance

- https://www.itilog.com/ provides Lat, Long of an address
- Create a bar chart with the absolute difference between the Earth and Euclidean distances for each capital
- Are the Earth and Euclidean distances coherent? Make a plot to show that

- Submit a (well-commented) python notebook
Homework 2.2

What is the most “central” EU capital, i.e., the one with the lowest average Earth distance with the other EU capitals?

- Create a bar chart with the average distance for each EU capital, sorted in increasing order
- Repeat the exercise for at least another continent
- Submit a (well-documented) python notebook
Homework 2.3

Find at least three examples of regular or irregular tessellations in natural or artificial ecosystems (other than those presented in class). Discuss the reason (if any) why the tiles have precisely that shape.

● Write a blog post (2-3 pages max.) about it
  ● Include the references you used (to papers, blog posts, newspaper articles, videos, or whatever)
Homework 2.4

Find objects (other than humans, animals, and transportation means) that may be described by trajectories. Do the same for flows.

- Write a blog post (2-3 pages max.) about it!
  - Include references (to papers, blog posts, newspaper articles, videos, or whatever)
Homeworks to be delivered by Friday, 29th 2022
Homework 2.5

Create your own trajectory! Track your movements for an entire week, tracking all points of interest you visited (e.g., home, friends’ home, university, supermarket, gym, bars).

- Use [https://www.itilog.com/](https://www.itilog.com/) to detect Lat & Long of places based on their address.
- Create a `TrajDataFrame` in scikit-mobility and visualize it.
  - If you want, add to `TrajDataFrame` the place’s name or address
- Submit the notebook and the data, or a link to them
Homework 2.6

Use scikit-mobility to create a squared tessellation over Florence (500m). For any pair of tiles, generate a random flow choosing a number uniformly at random in [0, 1000]. Repeat for h3 tessellation (500m) and a Voronoi tessellation (with n=100 random points).

- Create a FlowDataFrame and visualize it in skmob.
- Are the flows realistic? Why? Comment on it.
- Save the tessellations into a shapefile using GeoPandas

- Submit a (well-commented) notebook.
Homework 2.7

What is the theoretic upper bound of the mapping error of a trajectory of n points into a squared tessellation of size s meters? And what is this upper bound for a hexagonal or triangular tessellation?

- Use a real trajectory dataset and map the trajectory into a tessellation. Compute the error. Is it actually lower than the theoretic upper bound?

- Submit a (well-documented) notebook
Homework 2.8

Compute the area of Italian subdivisions using GeoPandas and plot their area as a bar chart

- Download a shapefile that describe Italian regions (e.g., here)
- Make a bar chart, put the regions in increasing order of area (put the region’s name on the x axis)
- Repeat for provinces and municipalities (plot only the 100 municipalities with the highest area)
- Plot the shape of each region (in blue), with the shape of its capital municipality (in red)
- Submit a (well-documented) notebook
Homework 2.9

Create and plot a `GeoDataFrame` with the top 1% and the bottom 1% municipalities in Italy based on their area.

- Download a shapefile that describe Italian regions (e.g., [here](#)).
- Create a `GeoDataFrame` with two rows (top 1% and bottom 1%) and the corresponding multipolygons.
- Plot the multipolygons with folium.
- Submit a (well-documented) notebook.
Create a GeometryCollection with Shapely regarding an imaginary squared Island with a lake, a house, and a road.

- Island: a square centered at (0, 0) with side 10
- Lake: rectangle base 3, height 2, bottom-left corner at (-2.5, 0)
- House: Point centered at (3, 2) with size (buffer) 1
- Road: line connecting the lake’s center and the house’s center
- Plot the collection using folium

Submit a (well-documented) notebook
Homework 2.11

Several "Los Pollos Hermanos" vans, carrying large quantities of methamphetamine (meth), were attacked by a drug cartel 10 times in an area in New Mexico. The DEA thinks the meth lab is at the centroid of this area.

- Compute the smallest polygon that contains all the points corresponding to the attacks
- Create a GeometryCollection that contains New Mexico, the polygon, and the points within it
- Visualize the collection and the centroid in folium (use markers for points, color the centroid differently)
- Randomly generate the attacks’ points in New Mexico
- Submit a (well-commented) notebook
Download the geojson files describing the world of “A Game of Thrones” from this repository. Plot the places mentioned in three chapters of your choice.

- Create some GeoDataFrames to store info and geometries of continents, islands, lakes, rivers, roads, the Wall, and the locations.
- Plot them using geopandas
- Select 3 chapters from here, and visualize all places mentioned in the chapters that are present in the locations GeoDataFrame.
- Plot each chapter’s points differently and make a legend with the Chapter number and title.
- Submit a (well-commented) notebook