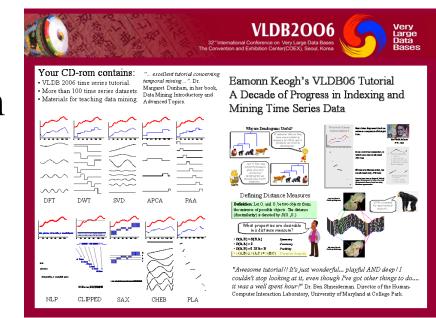
Introduction to Time Series Mining

Slides edited from Keogh Eamonn's tutorial:



25.1750 25.2250 25.2500 25.2500 25.2750 25.3250 25.3500 25.3500

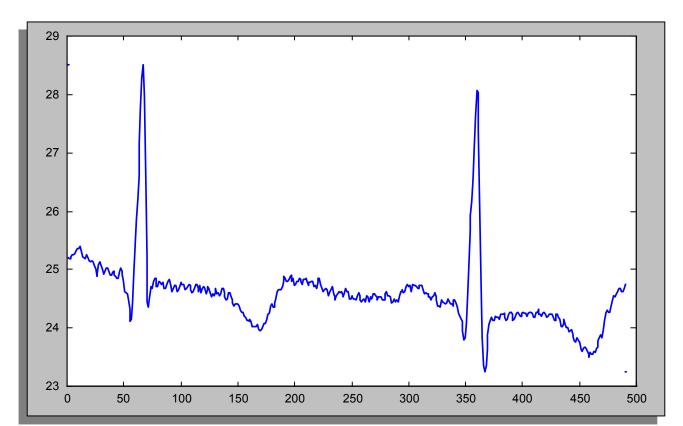
25.4000 25.4000 25.3250

25.2250 25.2000

25,1750

What are Time Series?

A time series is a collection of observations made sequentially in time.



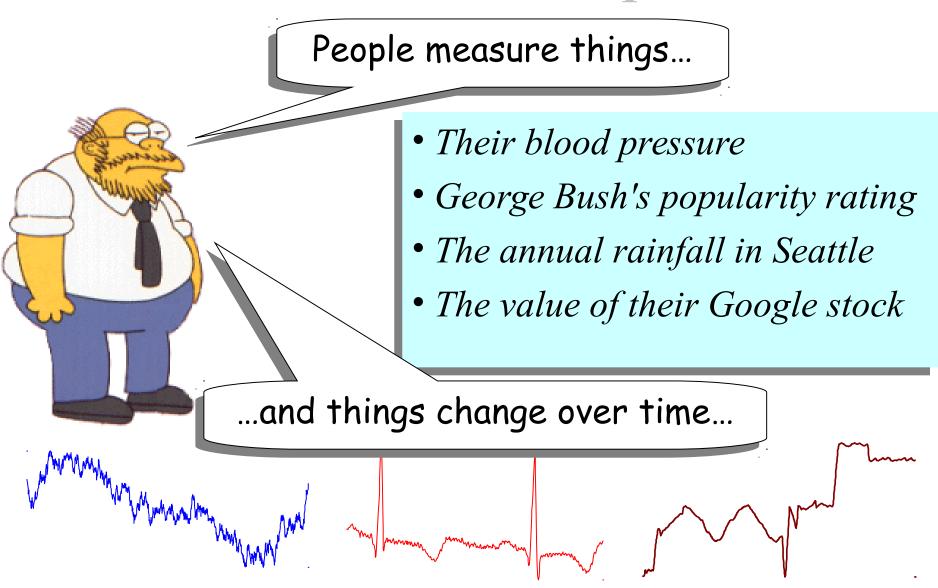


24.6250 24.6750 24.6750 24.6250 24.6250 24.6250

24,6750

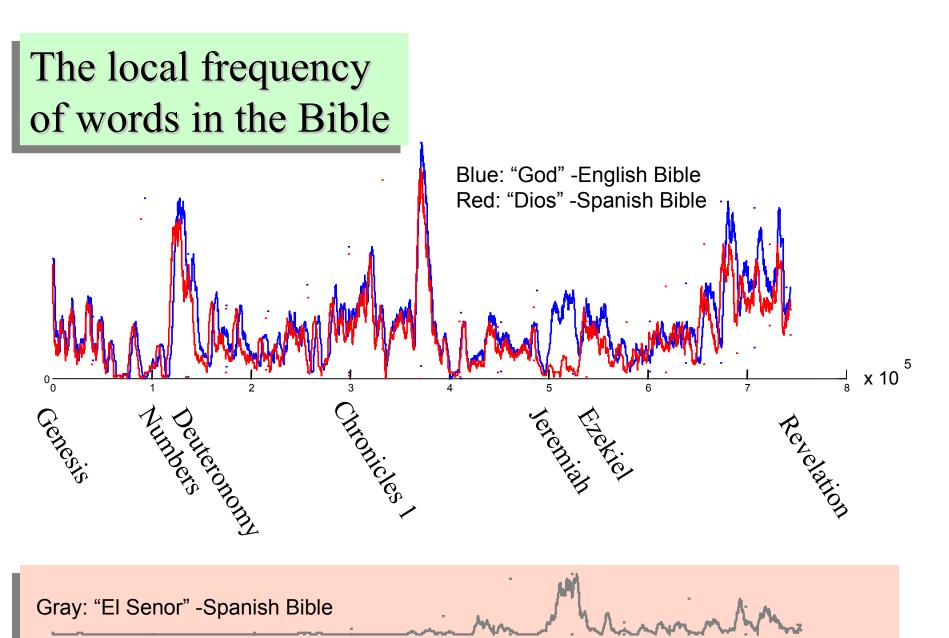
24.7500

Time Series are Ubiquitous! I



Thus time series occur in virtually every medical, scientific and businesses domain

Text data, may best be thought of as time series...



Video data, may best be thought of as time series...

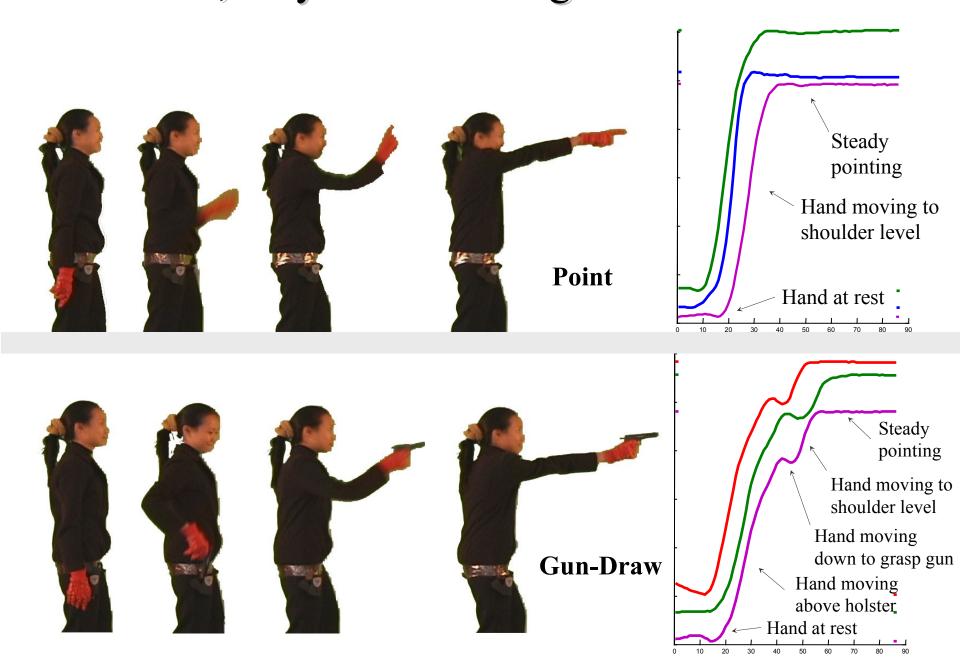
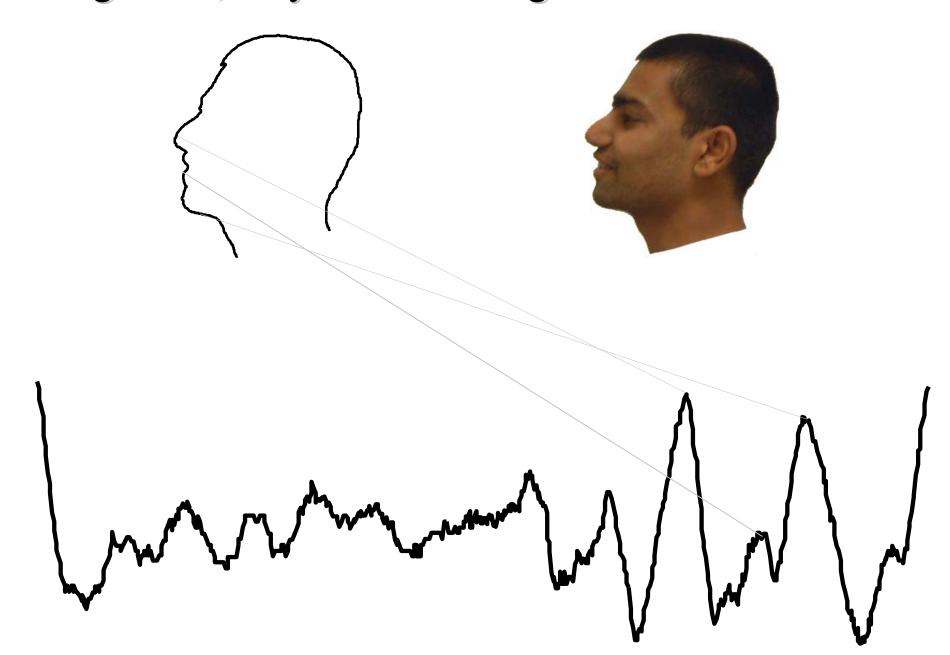
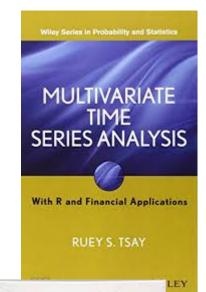


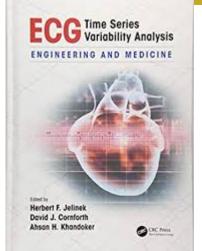
Image data, may best be thought of as time series...



A huge literature





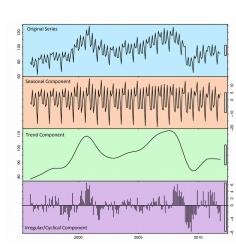


Classical topics on TS

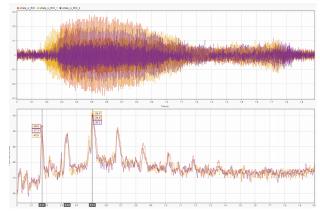
• Extracting trends, seasonal behaviours, etc.

Forecasting

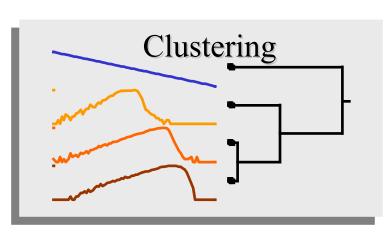


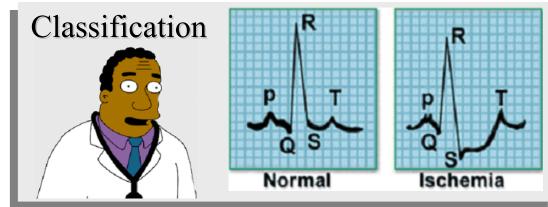


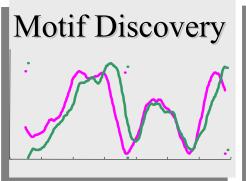
- Spectral analysis (frequencies)
 - e.g. signal processing

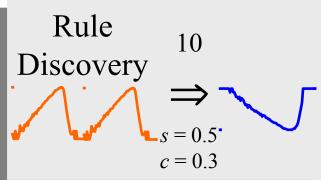


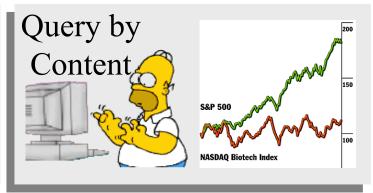
A database & Data Mining perspective

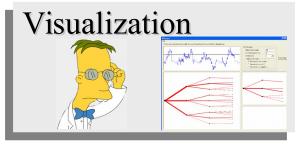


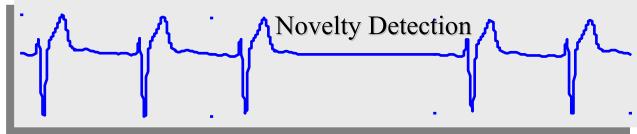








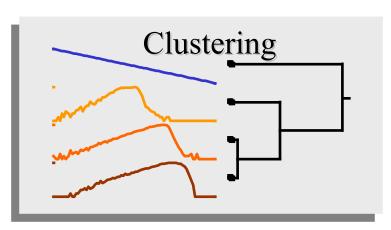


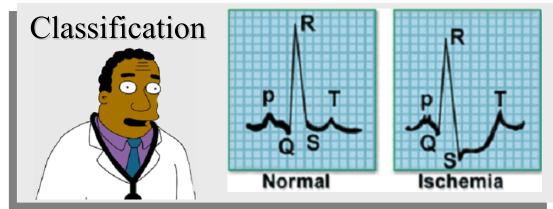


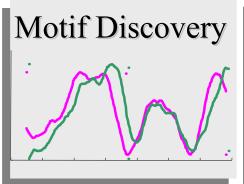
Our focus

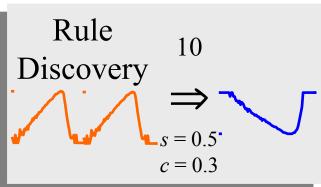
- Similarity-based tasks
 - Standard clustering, classification (KNN), etc.
- Outlier detection
- Frequent patterns (Motifs)

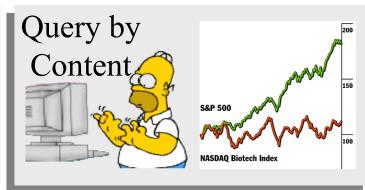
All these problems require similarity matching

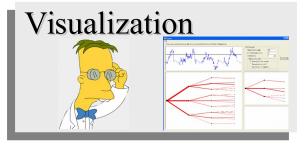


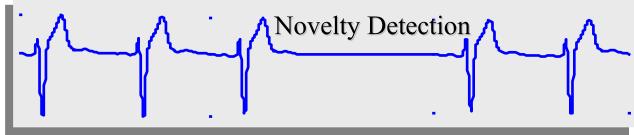












What is Similarity?

The quality or state of being similar; likeness; resemblance; as, a similarity of features. Webster's Dictionary

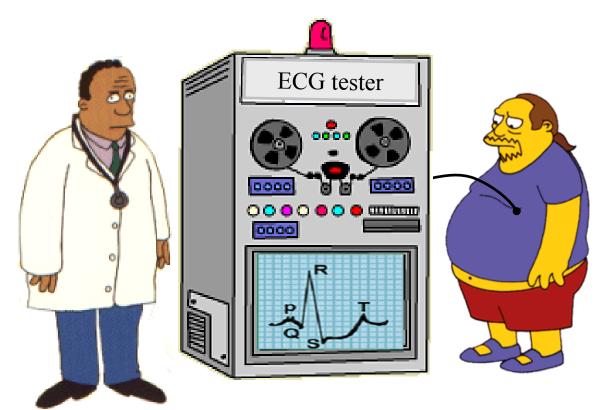


Similarity is hard to define, but...
"We know it when we see it"

The real meaning of similarity is a philosophical question.

We will take a more pragmatic approach.

Here is a simple motivation for the first part of the tutorial

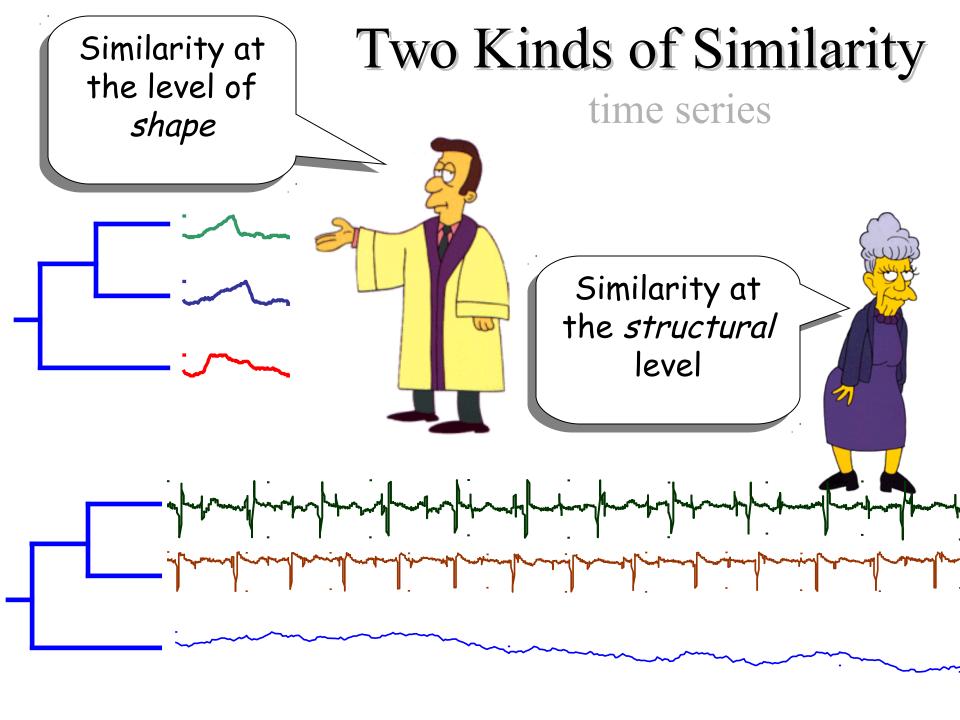


You go to the doctor because of chest pains. Your ECG looks strange...

You doctor wants to search a database to find **similar** ECGs, in the hope that they will offer clues about your condition...

Two questions:

- How do we define similar?
- How do we search quickly?



Euclidean Distance Metric

Given two time series:

$$Q = q_1 \dots q_n \qquad C = c_1 \dots c_n$$

$$D(Q,C) \equiv \sqrt{\sum_{i=1}^{n} (q_i - c_i)^2}$$

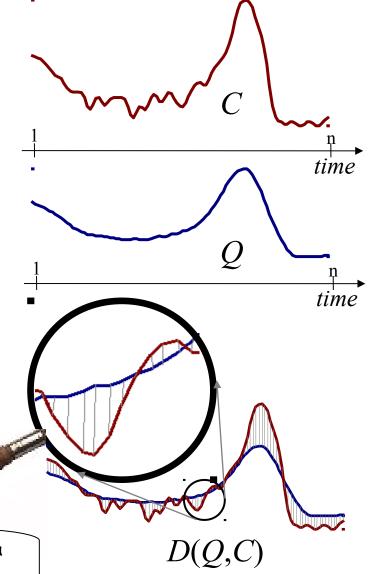
Same meaning as in transaction data:

- schema: <age, height, income, tenure>
- T1 = < 56, 176, 110, T2 = < 36, 126, 180,

 $D(T1,T2) = sqrt [(56-36)^2 + (176-126)^2 + (110-180)^2 + (95-80)^2]$



About 80% of published work in data mining uses Euclidean distance



Preprocessing the data before distance calculations



If we naively try to measure the distance between two "raw" time series, we may get very unintuitive results

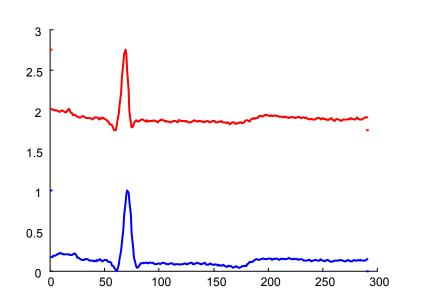
Euclidean distance is very sensitive to some "distortions" in the data. For most problems these distortions are not meaningful => should remove

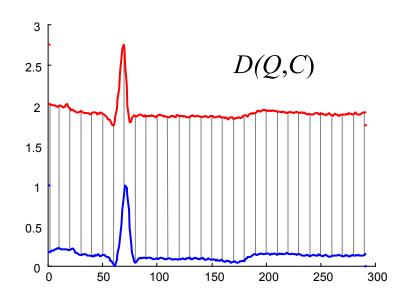


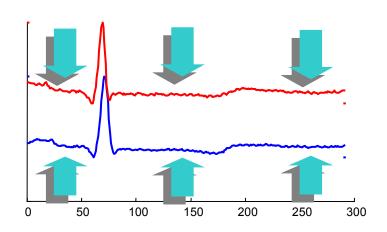
In the next few slides we will discuss the 4 most common distortions, and how to remove them

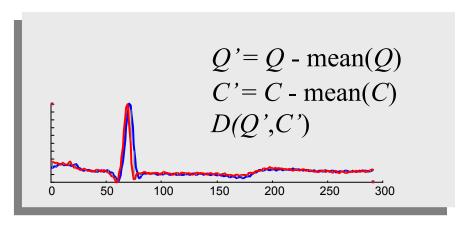
- Offset Translation
- Amplitude Scaling
- Linear Trend
- Noise

Transformation I: Offset Translation



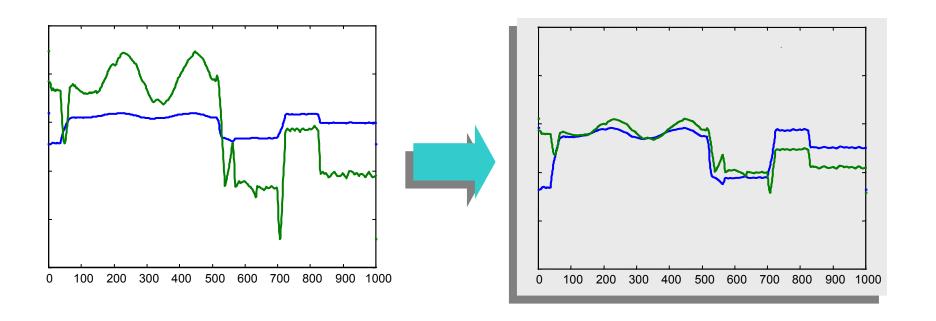






Remark: Q' and C' now have mean = 0

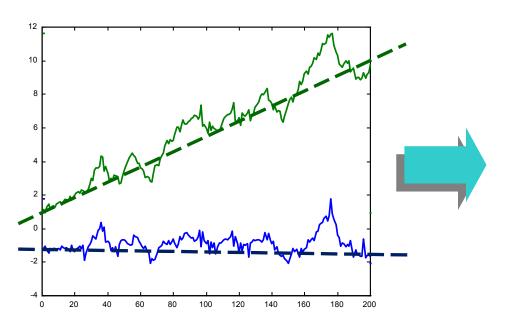
Transformation II: Amplitude Scaling

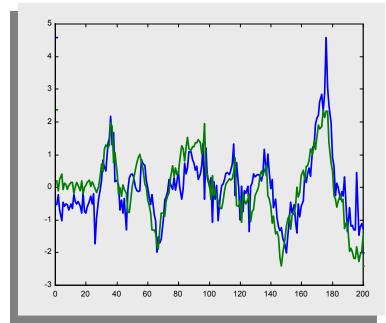


$$Q''=(Q-\operatorname{mean}(Q)) / \operatorname{std}(Q)$$

$$C''=(C-\operatorname{mean}(C)) / \operatorname{std}(C)$$
(i.e. zero-mean and one-std)
$$D(Q'',C'')$$

Transformation III: Linear Trend





Removing linear trend:

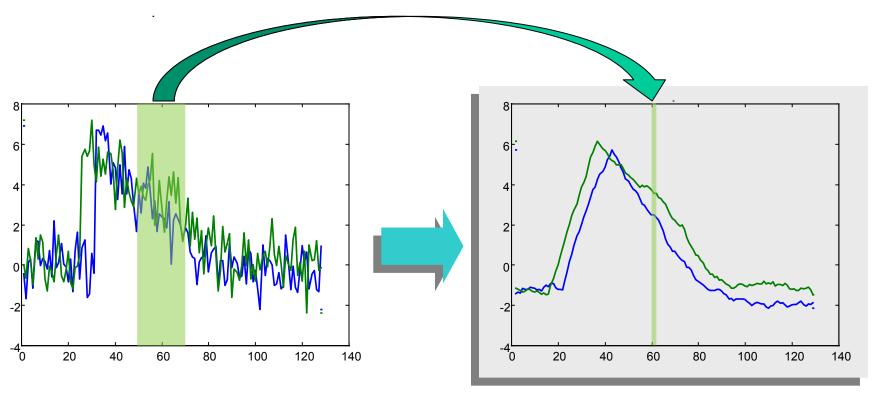
- •fit the best fitting straight line to the time series, then
- •subtract that line from the time series.

Removed linear trend

Removed offset translation

Removed amplitude scaling

Transformation IV: Noise



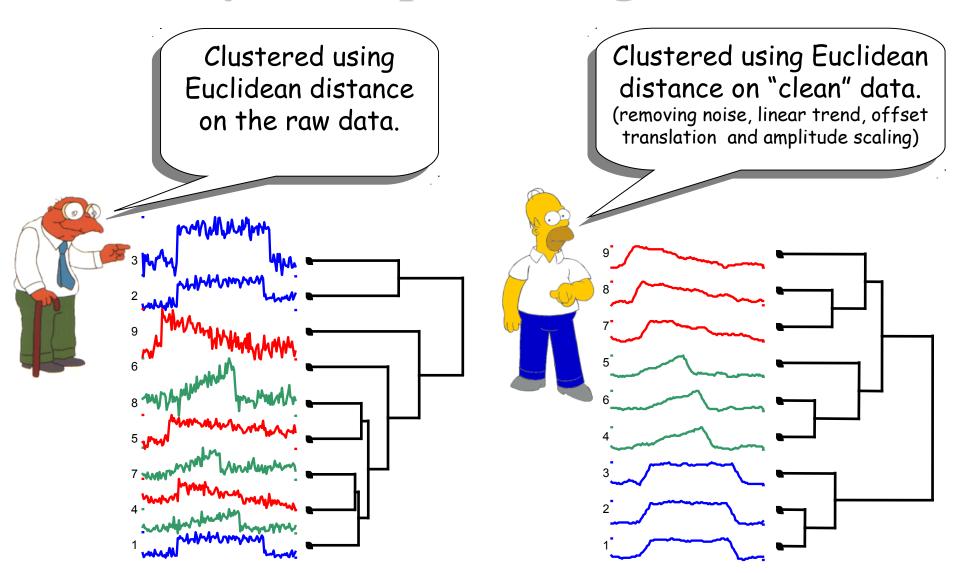
The intuition behind removing noise is...

Average each datapoints value with its neighbors.

$$Q' = \operatorname{smooth}(Q)$$

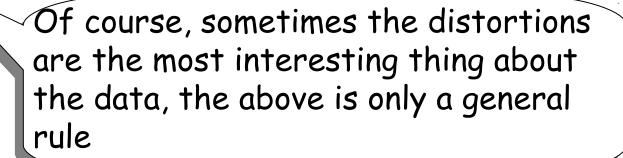
$$C' = \operatorname{smooth}(C)$$

A Quick Experiment to Demonstrate the Utility of Preprocessing the Data



Summary of Preprocessing

The "raw" time series may have distortions which we should remove before clustering, classification etc

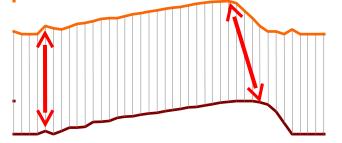




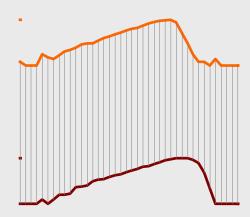
Beyond Euclidean: Dynamic Time Warping

• Sometimes two time series that are conceptually equivalent evolve at different speeds, at least in some

moments

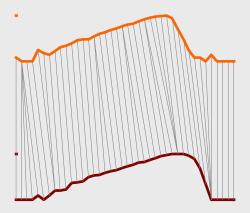


E.g. correspondence of peaks in two similar time series



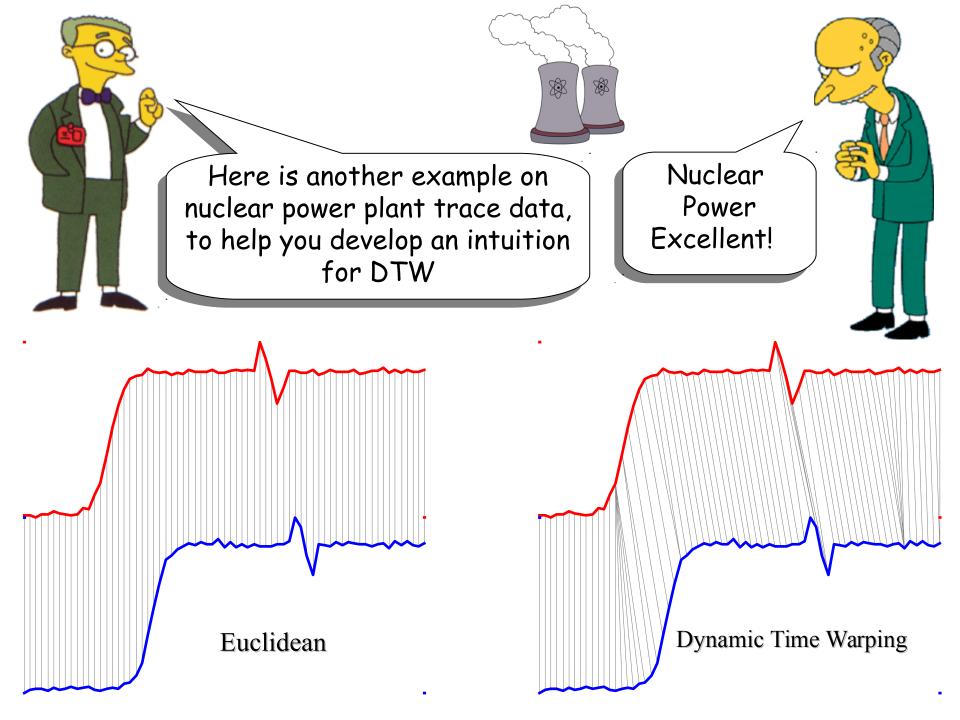
Fixed Time Axis

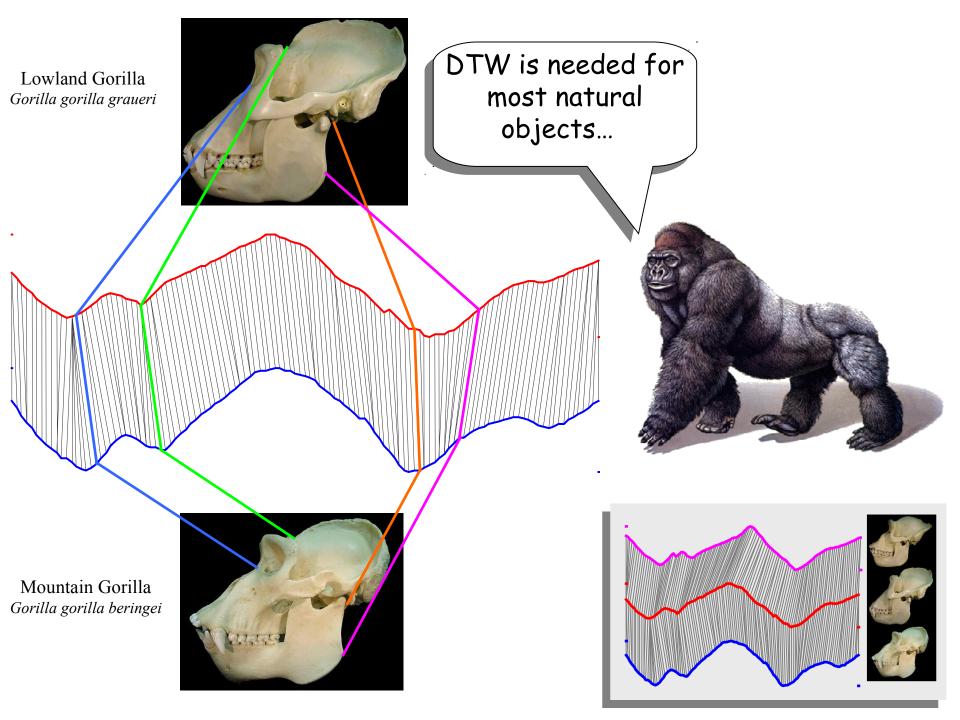
Sequences are aligned "one to one". Greatly suffers from the misalignment in data.



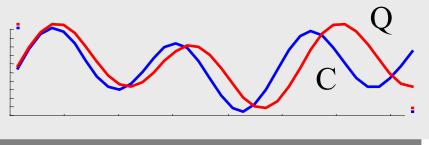
"Warped" Time Axis

Nonlinear alignments are possible. Can correct misalignments in data.





How is DTW Calculated? I



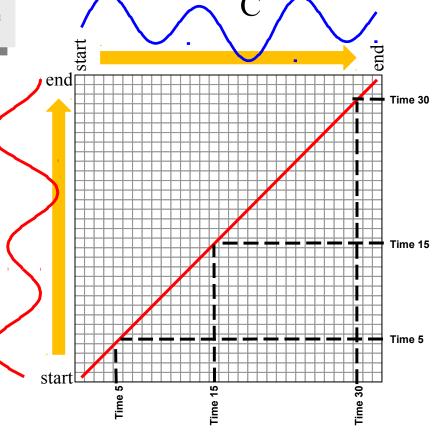
We create a matrix the size of | Q| by |C|, then fill it in with the distance between every pair of point in our two time series.

The **Euclidean distance** works only on the diagonal of the matrix

The sequence of comparisons performed:

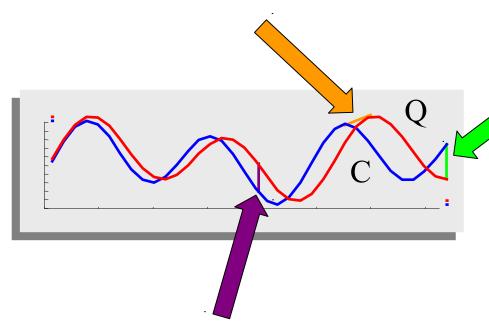
•Start from pair of points (0,0)

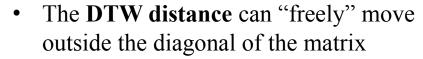
- •After point (i,i) move to (i+1,i+1)
- •End the process on (n,n)



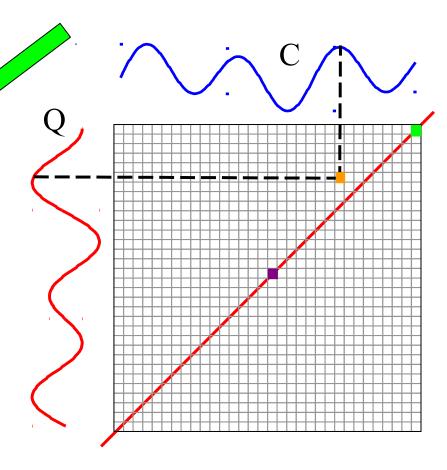
How is DTW Calculated? I

We create a matrix the size of | Q| by |C|, then fill it in with the distance between every pair of point in our two time series.





• Such cells correspond to temporally shifted points in the two time series



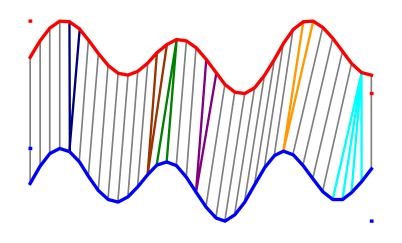
How is DTW Calculated? II

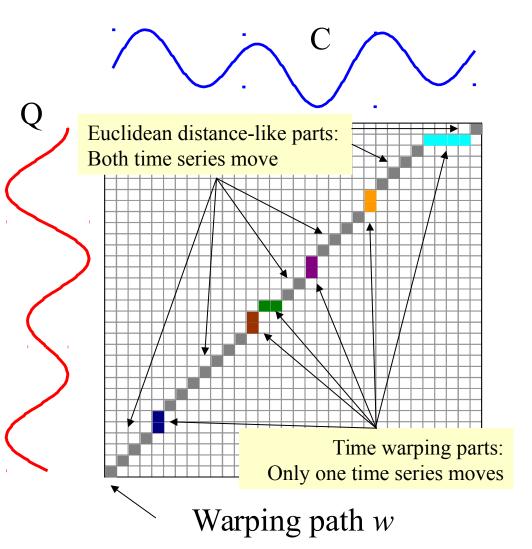
Every possible warping between two time series, is a path through the matrix.

The **DTW distance** can "freely" move outside the diagonal of the matrix

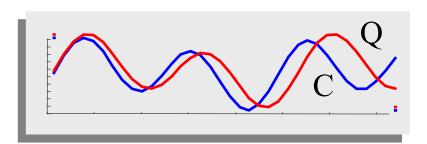
The only constraints:

- •Start from pair of points (0,0)
- •After point (i,j), either "i" or "j" increase by one, or both of them
- •End the process on (n,n)





How is DTW Calculated? II

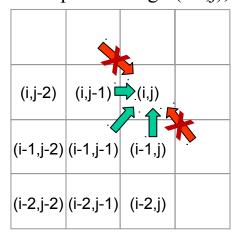


Definition of DTW as recursive function:

$$\gamma(i,j) = \text{cost of best path reaching cell } (i,j)$$

$$= d(q_i,c_i) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

Idea: best path must pass through (i-1,j), (i-1,j-1) or (i,j-1)



We want the best warping path:

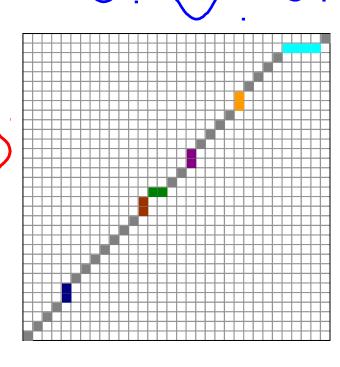


 $w_k = cost of the k-th points comparison$

$$\bullet W_k = |Qi - Cj|$$

$$(i,j)$$
 = position of w_k

$$\bullet w_k = [Qi - Cj] ^2$$



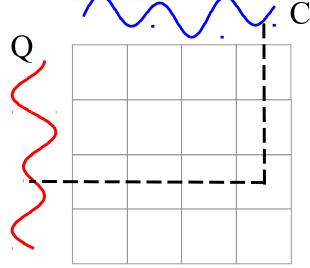
Remark: w can be longer than "n"

Dynamic programming approach

$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

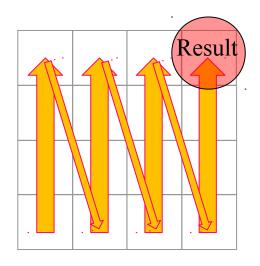
Step 1: compute the matrix of all $d(q_i,c_i)$

- •Point-to-point distances
- $\bullet D(i,j) = |Qi Cj|$



Step 2: compute the matrix of all path costs y(i,j)

- •Start from cell (1,1)
- •Compute (2,1), (3,1), ..., (n,1)
- •Repeat for columns 2, 3, ..., n
- •Final result in last cell computed



Dynamic programming approach

$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

Step 2: compute the matrix of all path costs y(i,j)

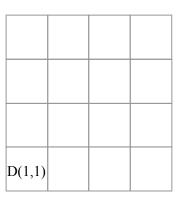
•Start from cell (1,1)

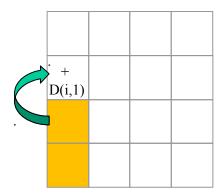
$$\begin{array}{rcl}
\Box & \gamma(1,1) & = & d(q_1,c_1) + \min\{ \gamma(0,0), \gamma(0,1), \gamma(1,0) \} \\
& = & d(q_1,c_1) \\
& = & D(1,1)
\end{array}$$

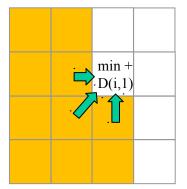
•Compute (2,1), (3,1), ..., (n,1)

$$\begin{array}{rcl}
\text{futc } (2,1), (3,1), \dots, (11,1) \\
& = \chi(i,1) = d(q_i,c_1) + \min\{ \gamma(i-1,0), \gamma(i-1,1), \gamma(i,0) \} \\
& = d(q_i,c_1) + \gamma(i-1,1) \\
& = D(i,1) + \gamma(i-1,1)
\end{array}$$

- •Repeat for columns 2, 3, ..., n
 - The general formula applies



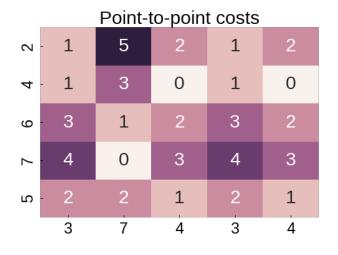


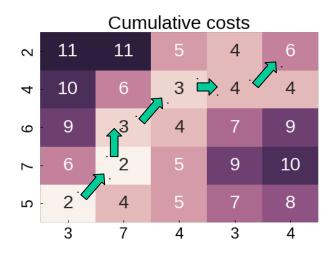


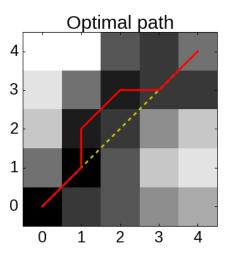
Dynamic programming approach

Example and How to infer the optimal path

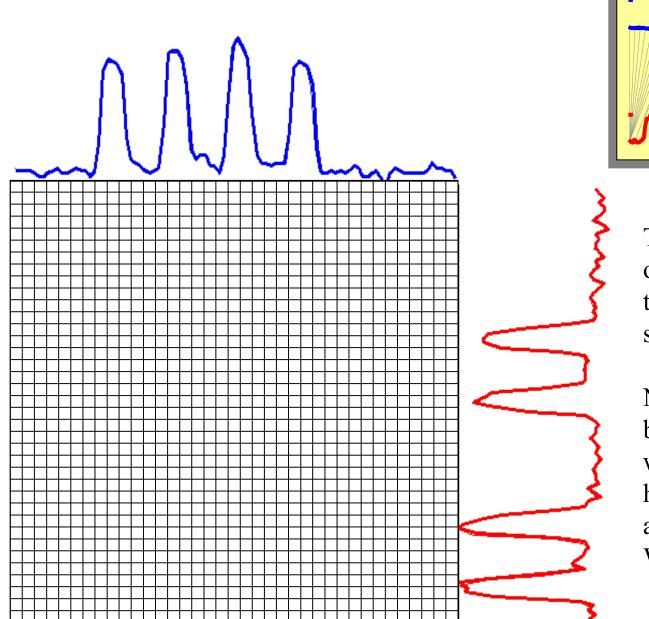
$$\gamma(i,j) = d(q_i,c_j) + \min\{ \gamma(i-1,j-1), \gamma(i-1,j), \gamma(i,j-1) \}$$

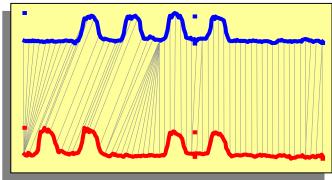






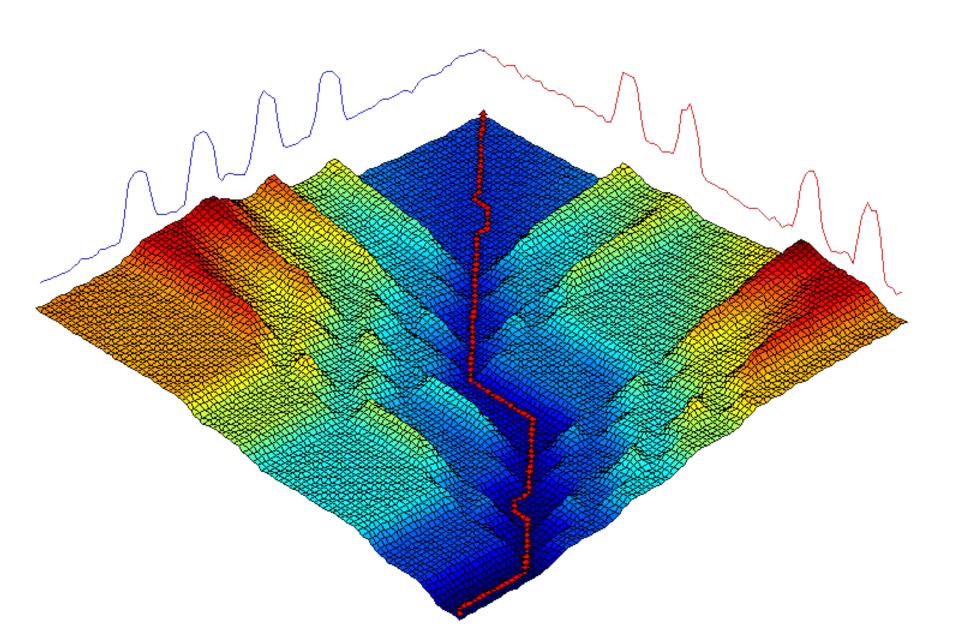
Let us visualize the cumulative matrix on a real world problem I



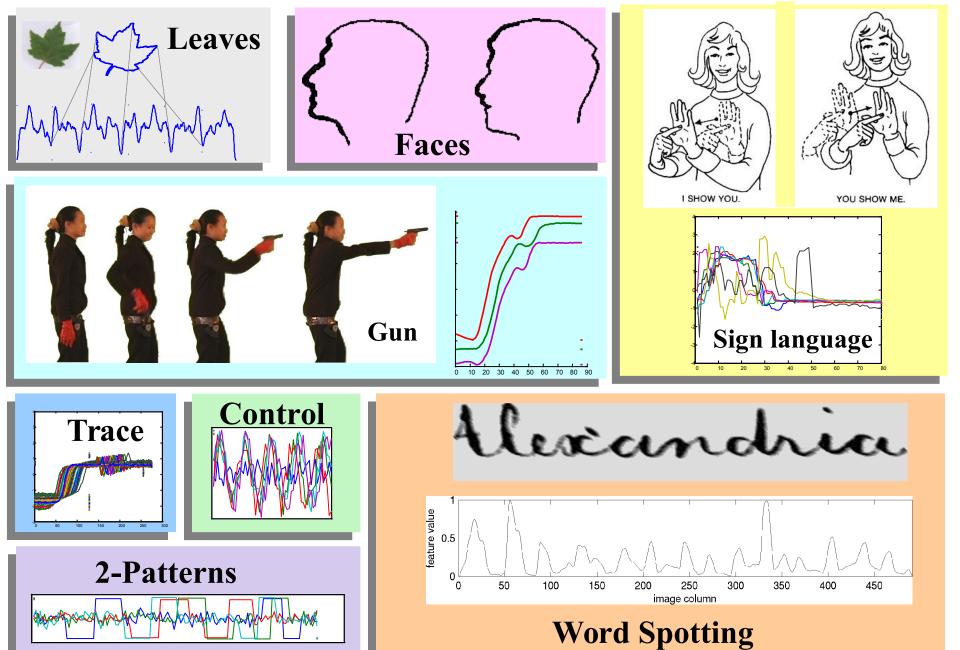


This example shows 2 one-week periods from the power demand time series.

Note that although they both describe 4-day work weeks, the blue sequence had Monday as a holiday, and the red sequence had Wednesday as a holiday. Let us visualize the cumulative matrix on a real world problem II



Let us compare Euclidean Distance and DTW on some problems



Results: Error Rate

Classification using 1-nearest-neighbor

•Class(x) = class of most similar training object

Leaving-one-out evaluation

•For each object: use it as test set, return overall average

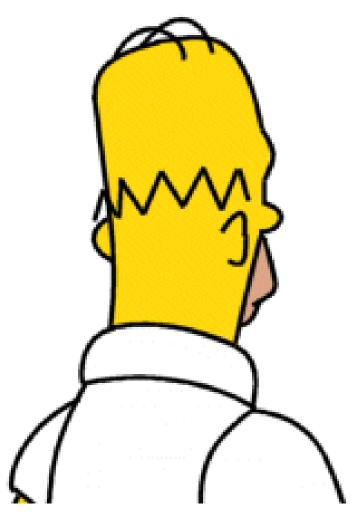
Dataset	Euclidean	DTW
Word Spotting	4.78	1.10
Sign language	28.70	25.93
GUN	5.50	1.00
Nuclear Trace	11.00	0.00
Leaves#	33.26	4.07
(4) Faces	6.25	2.68
Control Chart*	7.5	0.33
2-Patterns	1.04	0.00



Results: Time (msec)

Dataset	Euclidean	DTW	DTW is two
Word Spotting	40	8,600	orders of magnitude
Sign language	10	1,110	slower than Euclidean
GUN	60	11,820	197 distance
Nuclear Trace	210	144,470	687
Leaves	150	51,830	345
(4) Faces	50	45,080	901
Control Chart	110	21,900	199
2-Patterns	16,890	545,123	32

What we have seen so far...

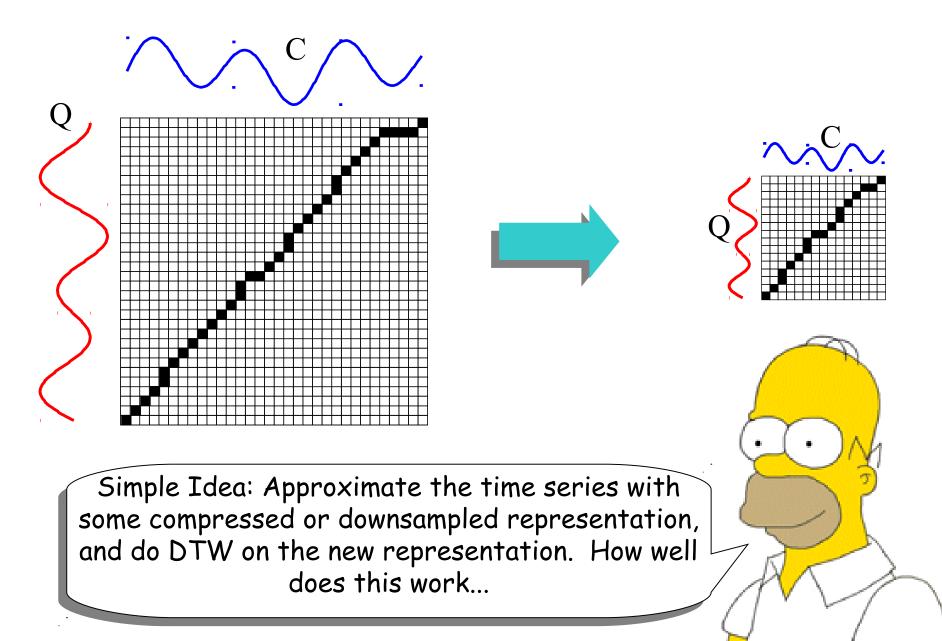


• Dynamic Time Warping gives **much better** results than Euclidean distance on virtually all problems.

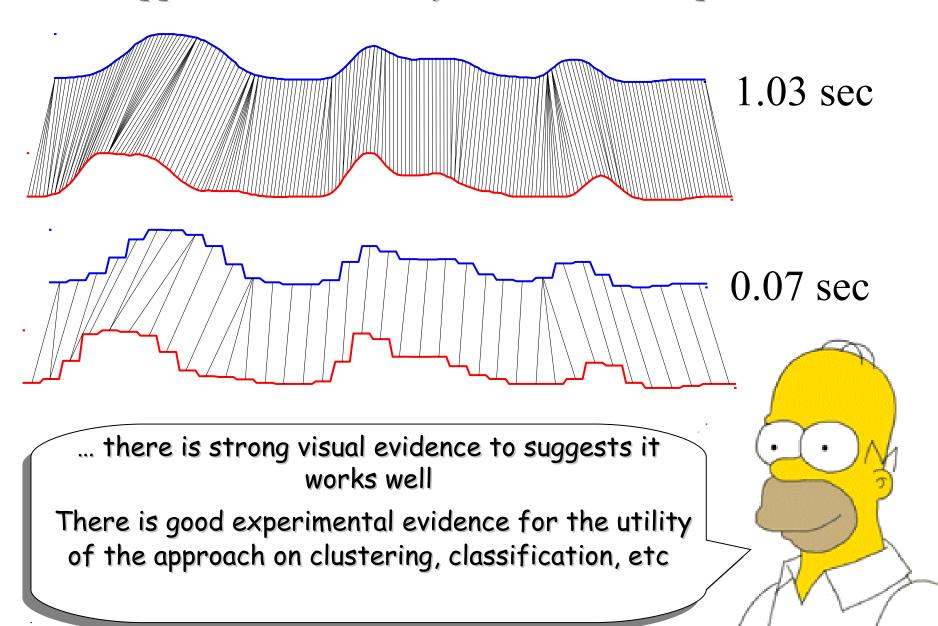
• Dynamic Time Warping is very very slow to calculate!

Is there anything we can do to speed up similarity search under DTW?

Fast Approximations to Dynamic Time Warp Distance I

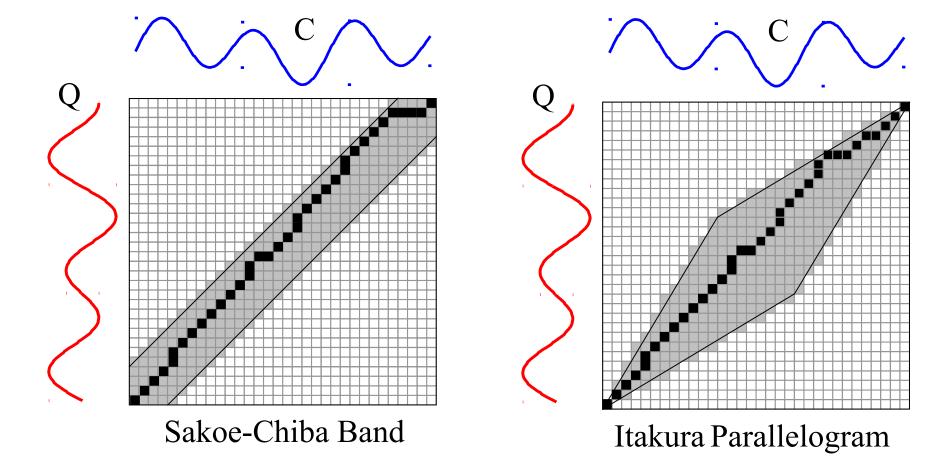


Fast Approximations to Dynamic Time Warp Distance II



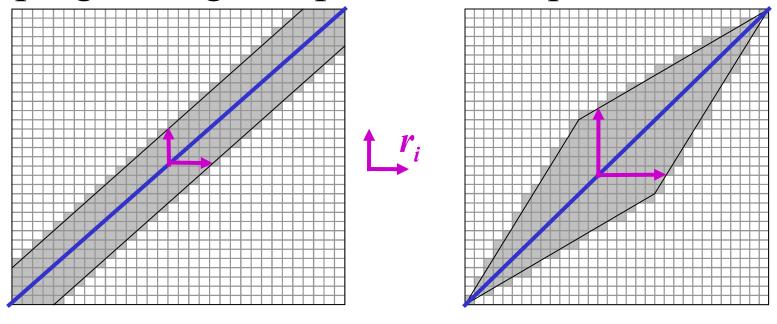
Global Constraints

- Slightly speed up the calculations
- Prevent pathological warpings



A global constraint constrains the indices of the warping path $w_k = (i,j)_k$ such that $j-r \le i \le j+r$

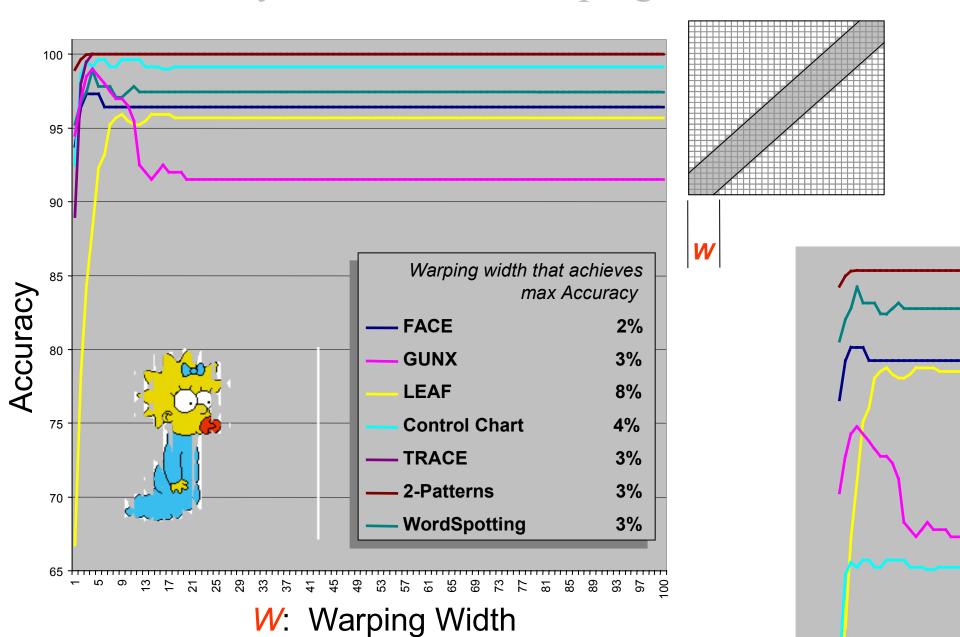
Where *r* is a term defining allowed range of warping for a given point in a sequence.

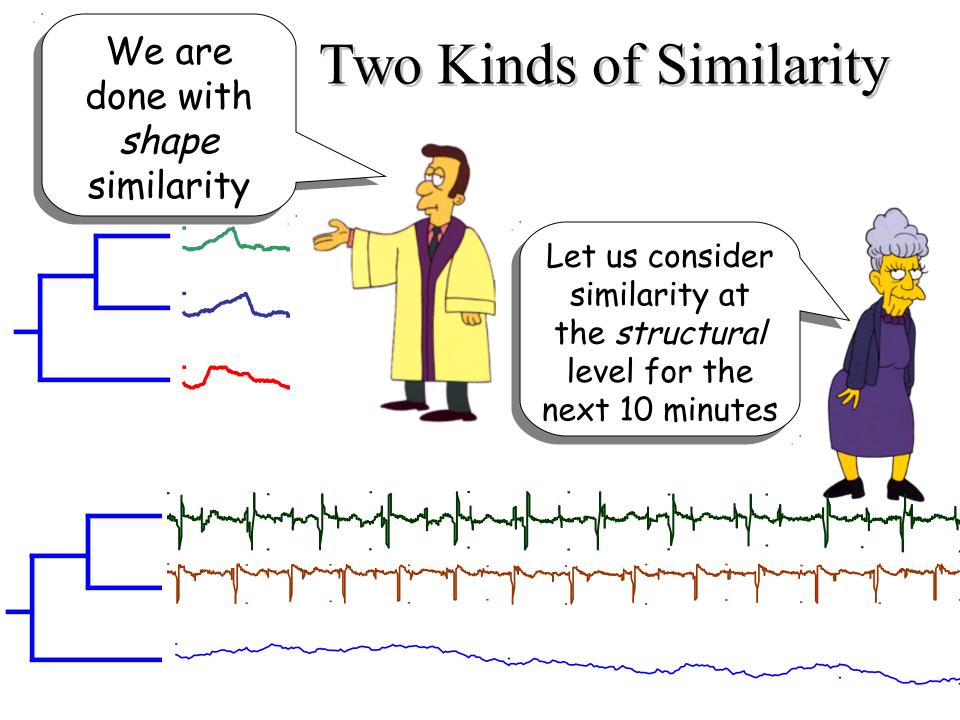


Sakoe-Chiba Band

Itakura Parallelogram

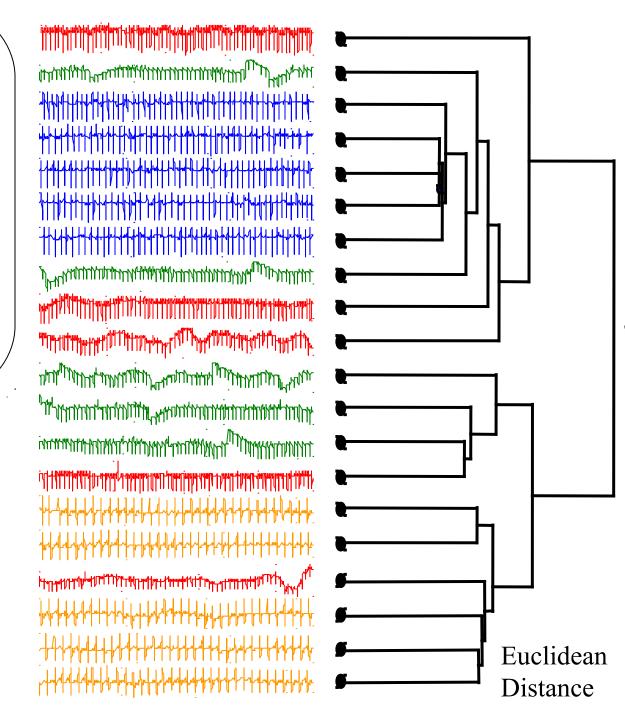
Accuracy vs. Width of Warping Window





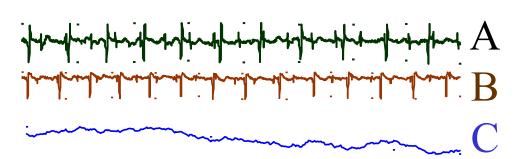
For long time series, shape based similarity will give very poor results. We need to measure similarly based on high level structure





Structure or Model Based Similarity

The basic idea is to extract global features from the time series, create a feature vector, and use these feature vectors to measure similarity and/or classify



Time Feature Series	A	В	C
Max Value	11	12	19
Autocorrelation	0.2	0.3	0.5
Zero Crossings	98	82	13
ARIMA	0.3	0.4	0.1



- features?
- distance measure/ learning algorithm?

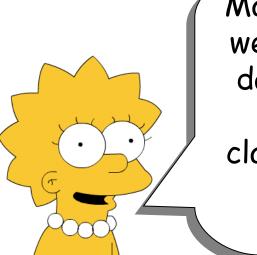
Feature-based Classification of Time-series Data

Nanopoulos, Alcock, and Manolopoulos

- features?
- distance measure/ learning algorithm?

Learning Algorithm

multi-layer perceptron neural network



Makes sense, but when we looked at the same dataset, we found we could be better classification accuracy with Euclidean distance!

Features

mean

variance

skewness

kurtosis

mean (1st derivative)

variance (1st derivative)

skewness (1st derivative)

kurtosis (1st derivative)

Learning to Recognize Time Series: Combining ARMA Models with Memory-Based Learning

Deng, Moore and Nechyba

- features?
- distance measure/ learning algorithm?

Distance Measure

Euclidean distance (between coefficients)

- Use to detect drunk drivers!
- Independently rediscovered and generalized by Kalpakis et. al. and expanded by Xiong and Yeung

Features

The parameters of the Box Jenkins model.

More concretely, the coefficients of the ARMA model.

"Time series must be invertible and stationary"

Deformable Markov Model Templates for Time Series Pattern Matching Ge and Smyth

- features?
- distance measure/ learning algorithm?

Distance Measure

"Viterbi-Like" Algorithm

Variations independently developed by Li and Biswas, Ge and Smyth, Lin, Orgun and Williams etc

There tends to be lots of parameters to tune...

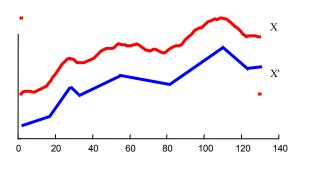
	A	В	С
A	0.1	0.4	0.5
В	0.4	0.3	0.3
С	0.5	0.2	0.3

Part 1

Features

The parameters of a Markov Model

The time series is first converted to a piecewise linear model



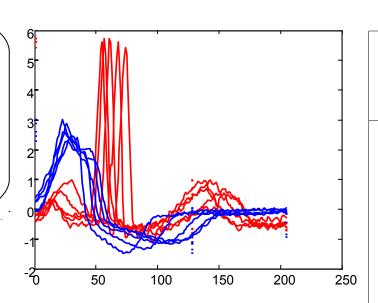






Deformable Markov Model Templates for Time Series Pattern Matching Ge and Smyth

On this problem the approach gets 98% classification accuracy*...



Part 2

Features

The parameters of a Markov Model

The time series is first converted to a piecewise linear model



But Euclidean distance gets 100%! And has no parameters to tune, and is tens of thousands times faster...



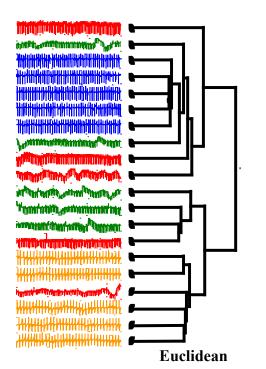
Compression Based Dissimilarity

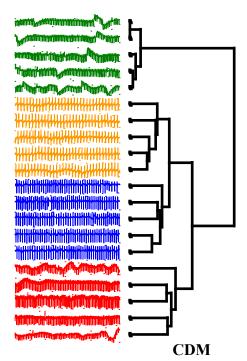
(In general) Li, Chen, Li, Ma, and Vitányi: (For time series) Keogh, Lonardi and Ratanamahatana

- features?
- distance measure/ learning algorithm?

Distance Measure

Co-Compressibility
$$CDM(x,y) = \frac{C(xy)}{C(x) + C(y)}$$





Features

Whatever structure the compression algorithm finds...

The time series is first converted to the SAX symbolic representation*

Compression Based Dissimilarity

Reel 2: Tension

Reel 2: Angular speed

Koski ECG: Fast 2

Koski ECG: Fast 1

Koski ECG: Slow 2

Koski ECG: Slow 1

Dryer hot gas exhaust

Dryer fuel flow rate

Ocean 2
Ocean 1

Evaporator: vapor flow

Evaporator: feed flow

Furnace: cooling input

Furnace: heating input

Great Lakes (Ontario)

Great Lakes (Erie)

Buoy Sensor: East Salinity

Buoy Sensor: North Salinity

Sunspots: 1869 to 1990

Sunspots: 1749 to 1869

Exchange Rate: German Mark

Exchange Rate: Swiss Franc

Foetal ECG thoracic

Foetal ECG abdominal

Balloon2 (lagged)

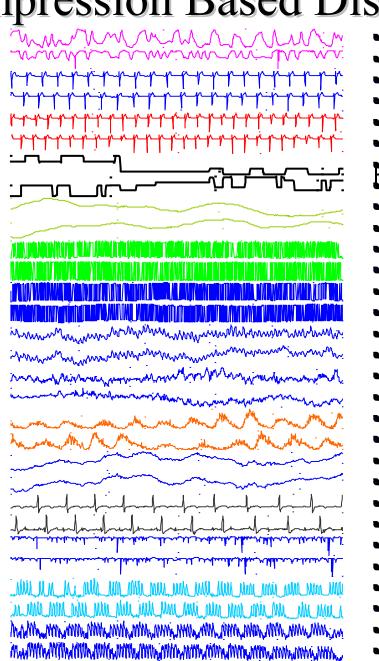
Balloon1

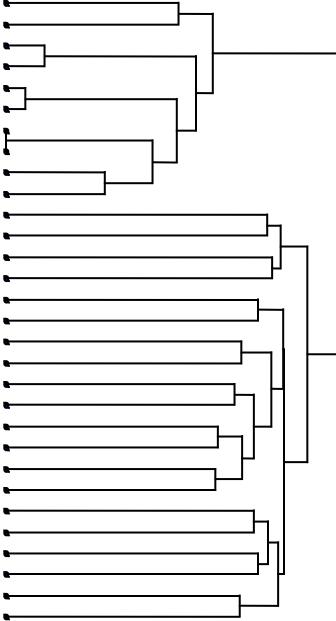
Power: April-June (Dutch)

Power: Jan-March (Dutch)

Power: April-June (Italian)

Power: Jan-March (Italian)





Summary of Time Series Similarity

- If you have *short* time series
 - use DTW after searching over the warping window size
- If you have *long* time series,
 - •and you know nothing about your data => try compression based dissimilarity
 - •if you do know something about your data => extract features

Time series analysis tasks

- Similarity-based tasks
 - Standard clustering, classification (KNN), etc.
- Outlier detection
- Frequent patterns (Motifs)
- Prediction

Anomaly (interestingness) detection

We would like to be able to discover surprising (unusual, interesting, anomalous) patterns in time series.

Note that we don't know in advance in what way the time series might be surprising

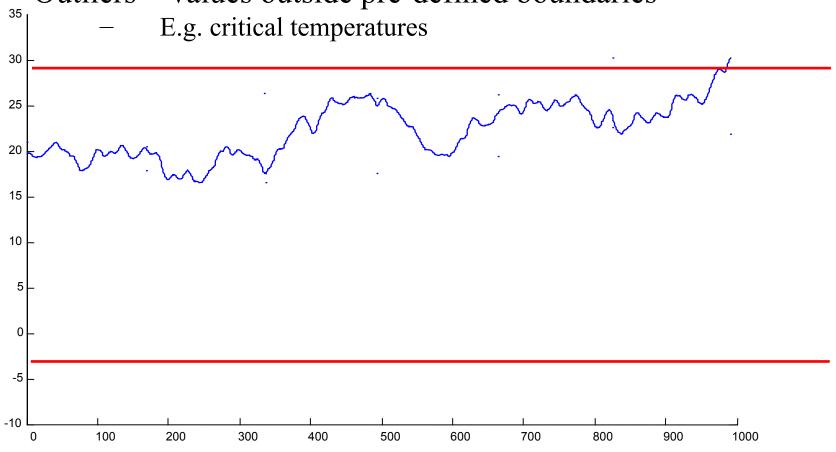
Also note that "surprising" is very context dependent, application dependent, subjective etc.



Simple Approaches I

Limit Checking

•Outliers = values outside pre-defined boundaries

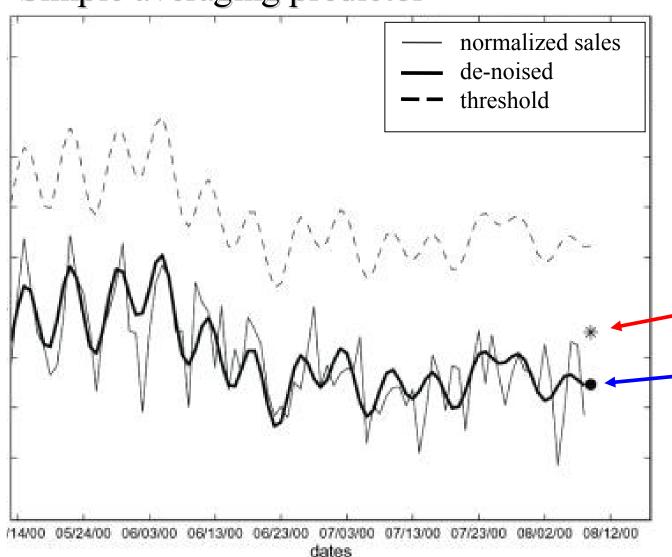


Simple Approaches II

Discrepancy Checking

Outliers = values far from those predicted by a model = values were unpredictable E.g. significant deviation from recent trend -5 -10

Discrepancy Checking: Example Simple averaging predictor



Early statistical detection of anthrax outbreaks by tracking over-the-counter medication sales

Goldenberg, Shmueli, Caruana, and Fienberg

Actual value

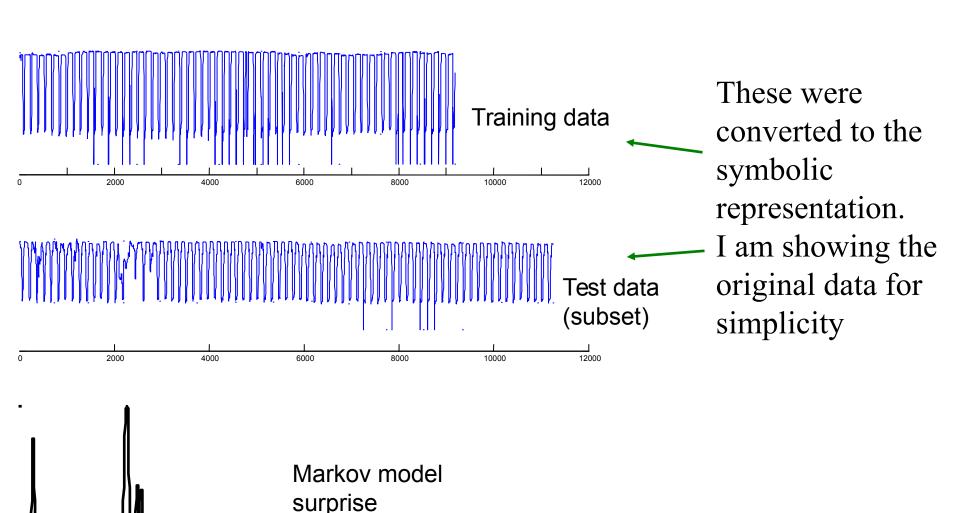
Predicted value

The actual value is greater than the predicted value, but still less than the threshold, so no alarm is sounded

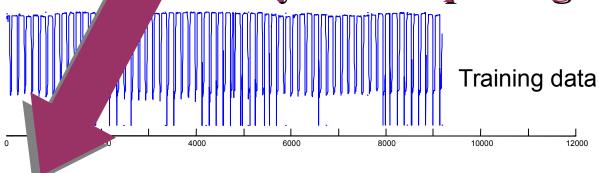
Discrepancy Checking with Markov models

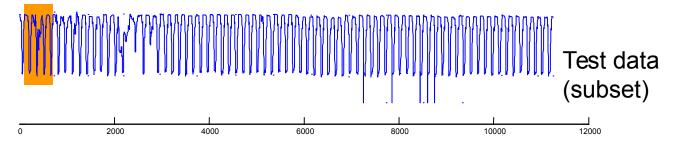
- Typical approach for text strings:
 - Take a set of text which has been labeled "normal"
 - Learn a Markov model for it
 - Any future data that is not modeled well by the Markov model you annotate as surprising.
- Time series can be easily converted to text
 - Discretization of numerical values
- We can use Markov models to find surprises in time series...

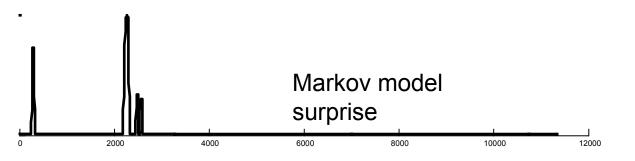
Discrepancy Checking with Markov models

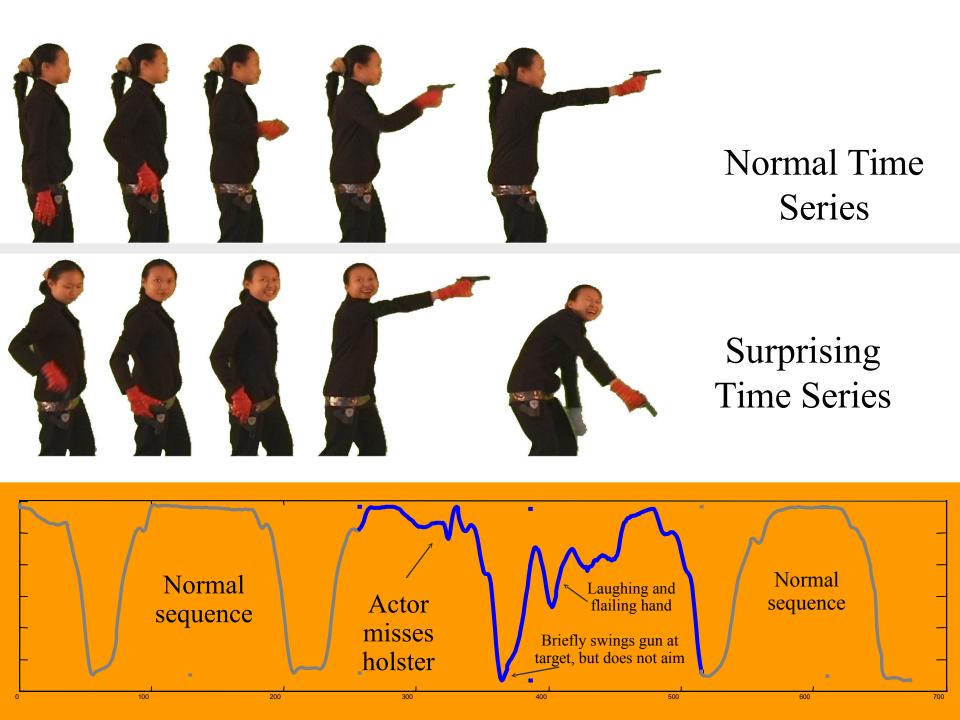












Anomaly (interestingness) detection

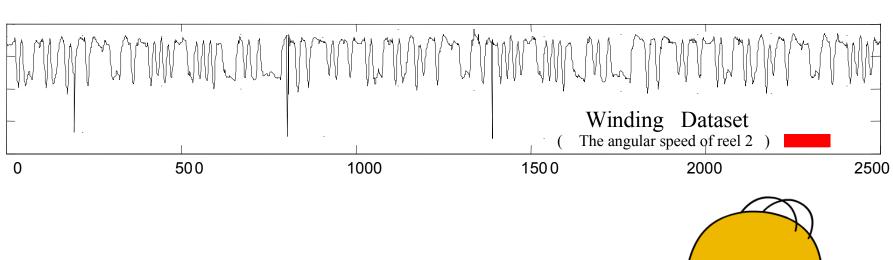
In spite of the nice example in the previous slide, the anomaly detection problem is wide open.

How can we find interesting patterns...

- Without (or with very few) false positives...
- In truly massive datasets...
- In the face of concept drift...
- With human input/feedback...
- With annotated data...

Time Series Motif Discovery

(finding repeated patterns)

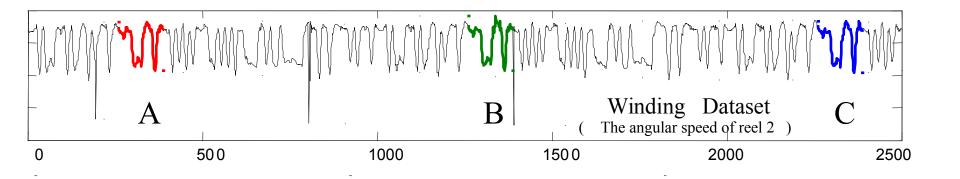


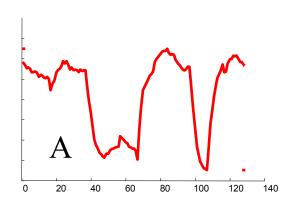
Are there any repeated patterns, of about this length — in the above time series?

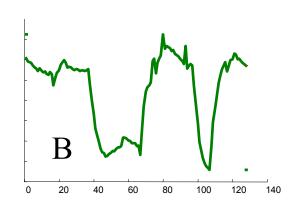


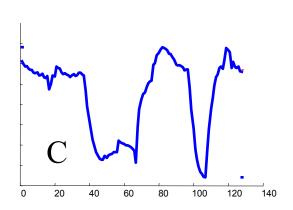
Time Series Motif Discovery

(finding repeated patterns)



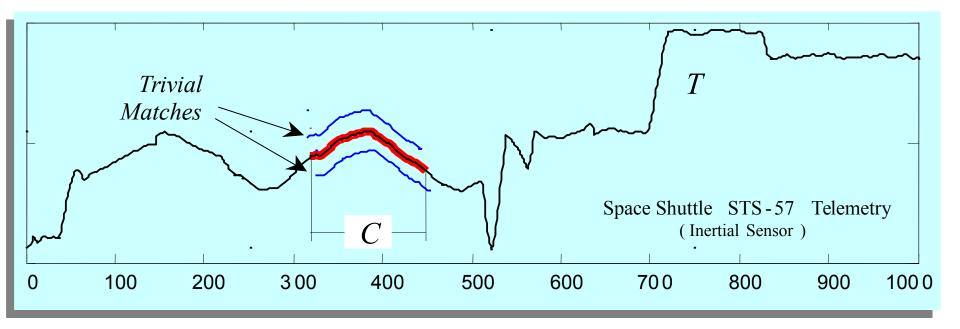






Why Find Motifs?

- · Mining **association rules** in time series requires the discovery of motifs. These are referred to as *primitive shapes* and *frequent patterns*.
- · Several time series **classification algorithms** work by constructing typical prototypes of each class. These prototypes may be considered motifs.
- · Many time series **anomaly/interestingness detection** algorithms essentially consist of modeling normal behavior with a set of typical shapes (which we see as motifs), and detecting future patterns that are dissimilar to all typical shapes.
- · In **robotics**, Oates et al., have introduced a method to allow an autonomous agent to generalize from a set of qualitatively different *experiences* gleaned from sensors. We see these "*experiences*" as motifs.
- · In **medical data mining**, Caraca-Valente and Lopez-Chavarrias have introduced a method for characterizing a physiotherapy patient's recovery based of the discovery of *similar patterns*. Once again, we see these "*similar patterns*" as motifs.
- Animation and video capture... (Tanaka and Uehara, Zordan and Celly)



Definition 1. *Match*: Given a positive real number R (called *range*) and a time series T containing a subsequence C beginning at position p and a subsequence M beginning at q, if $D(C, M) \le R$, then M is called a *matching* subsequence of C.

Definition 2. Trivial Match: Given a time series T, containing a subsequence C beginning at position p and a matching subsequence M beginning at q, we say that M is a trivial match to C if either p = q or there does not exist a subsequence M' beginning at q' such that D(C, M') > R, and either q < q' < p or p < q' < q.

Definition 3. *K-Motif*(n,R): Given a time series T, a subsequence length n and a range R, the most significant motif in T (hereafter called the I-Motif(n,R)) is the subsequence C_1 that has highest count of non-trivial matches (ties are broken by choosing the motif whose matches have the lower variance). The Kth most significant motif in T (hereafter called the K-Motif(n,R)) is the subsequence C_K that has the highest count of non-trivial matches, and satisfies $D(C_K, C_V) > 2R$, for all $1 \le i \le K$.

OK, we can define motifs, but how do we find them?

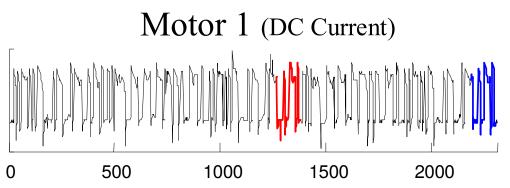
The obvious brute force search algorithm is just too slow...

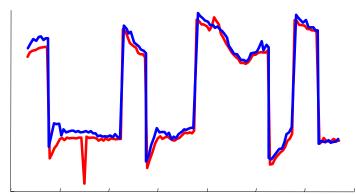
Many algorithms realized, main ones are based on *random projection** and SAX discretized representation of times series (useful to compute quick lower bound distances).

* J Buhler and M Tompa. Finding motifs using random projections. In RECOMB'01. 2001.

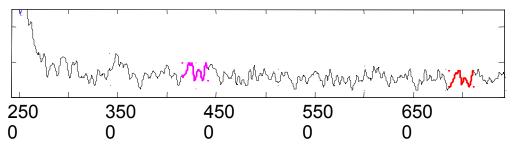


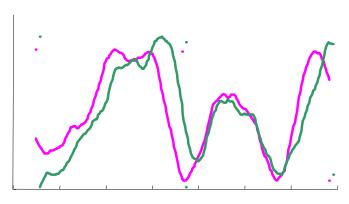
Some Examples of Real Motifs







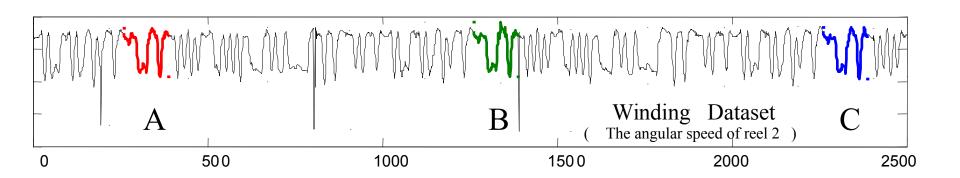




Motifs Discovery Challenges

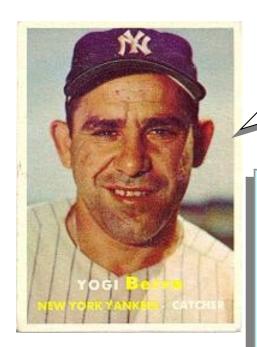
How can we find motifs...

- Without having to specify the length/other parameters
- In massive datasets
- While ignoring "background" motifs (ECG example)
- Under time warping, or uniform scaling
- While assessing their significance



Finding these 3 motifs requires about 6,250,000 calls to the Euclidean distance function

Time Series Prediction



Yogi Berra 1925 - 2015

Prediction is hard, especially about the future

There are two kinds of time series prediction

- **Black Box**: Predict tomorrows electricity demand, given *only* the last ten years electricity demand.
- White Box (side information): Predict tomorrows electricity demand, given the last ten years electricity demand *and* the weather report, *and* the fact that fact that the world cup final is on and...

Black Box Time Series Prediction

- Very difficult task, at least in longer-term prediction
- Several solutions are claiming great results, yet they are mostly still unreliable:
 - A paper in SIGMOD 04 claims to be able to get better than 60% accuracy on black box prediction of financial data (random guessing should give about 50%). The authors later agreed to test blind on a external and again got more than 60% -- though it was completely random data (quantum-mechanical random walk)...
 - A paper in SIGKDD in 1998 did black box prediction using association rules, more than twelve papers extended the work... but then it was proved that the approach *could* not work*!

End of Time series module

