Humanoid Robots and biped locomotion

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Outline

- What is a Humanoid?
- Why Develop Humanoids?
- Challenges in Humanoid robotics
- Active vs Passive Locomotion
- Active Bipedal Locomotion
  - Humanoid Robot kinematics
  - Humanoid Robot Dynamics (Stability via ZMP)

What is a Humanoid?

- Humanoid refers to any being whose body structure resembles that of a human: head, torso, legs, arms, hands.

- But it is also a robot made to resemble a human both in **appearance** and **behavior**.

- The difference between a **robot** and **android** is only skin-deep, looks exactly like humans on the outside, but with internal mechanics of humanoid robot.

Why Develop Humanoids?

- More rational reasons
  - They can work in human environment without a need to adapt themselves or to change the environment
  - Our environment and our tools are adapted for us
  - Why adapt all to robots?!
 - It is easier for a human being to interact with a human-like being

Challenges in Humanoids

- Bipedal human-like locomotion
  - Stable gait
    - Changing model during one/two feet support walking
    - Two legs, two arms, head, torso
- Hyper DOF system (>20)
  - Complex kinematics and dynamics
- Complex real-time control architecture

Active vs. Passive Locomotion

- Common humanoid uses all their DOF to perform the movement:
  - Continuous motor consumption (including arms)
  - Continuous motor control and synchronization
  - Extremely complex real-time control
- How is possible to reduce complexity?
  - Reducing number of active DOF
  - Using DOF only when it is strictly necessary
  - Using energy of previous step to generate the next
  - These actions reduce also the consumption
  - Robot not fully controllable

Kinematics of Humanoid Robots
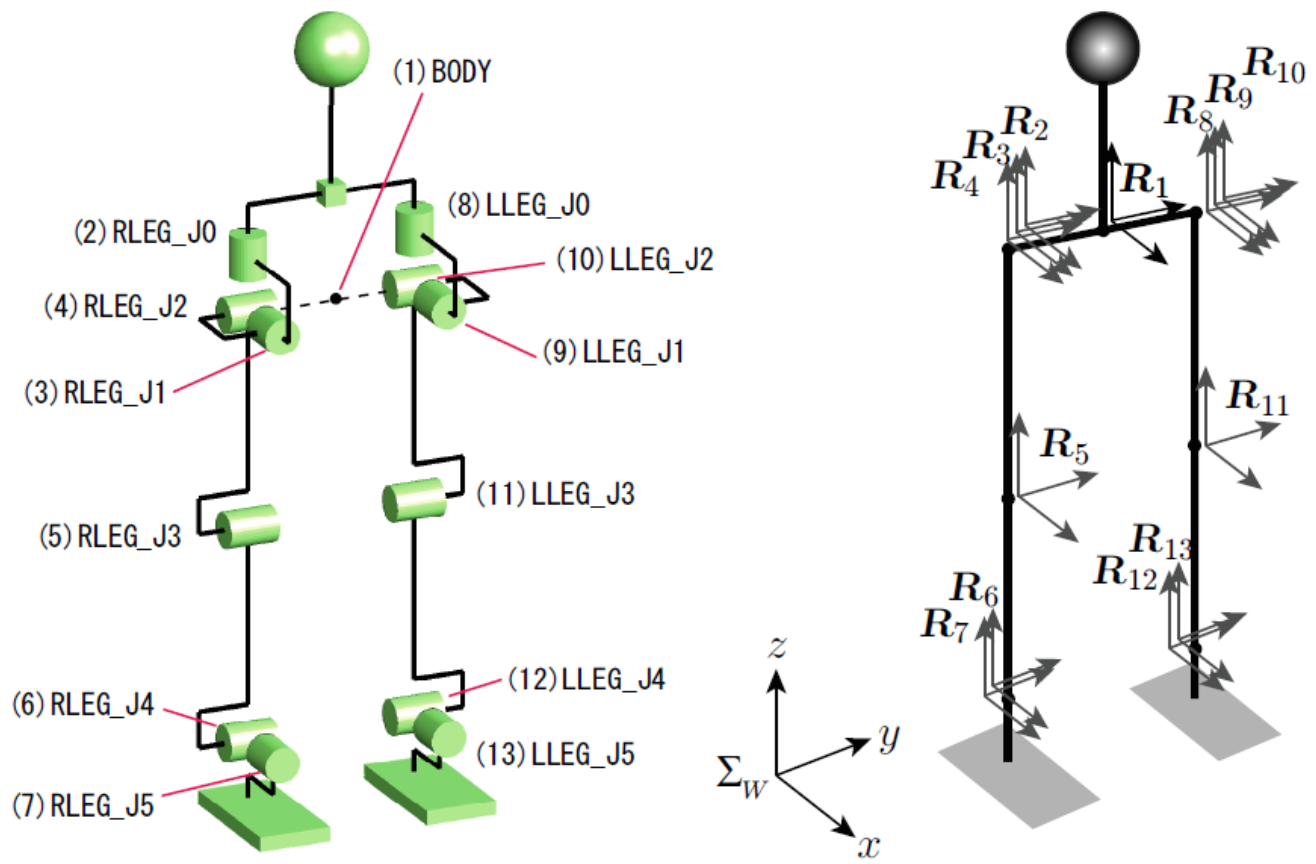
Kinematics of Humanoid Robots (II)

- Numerical solution to Inverse Kinematics
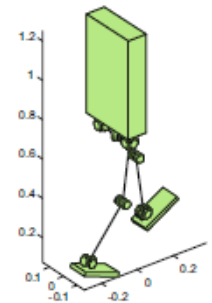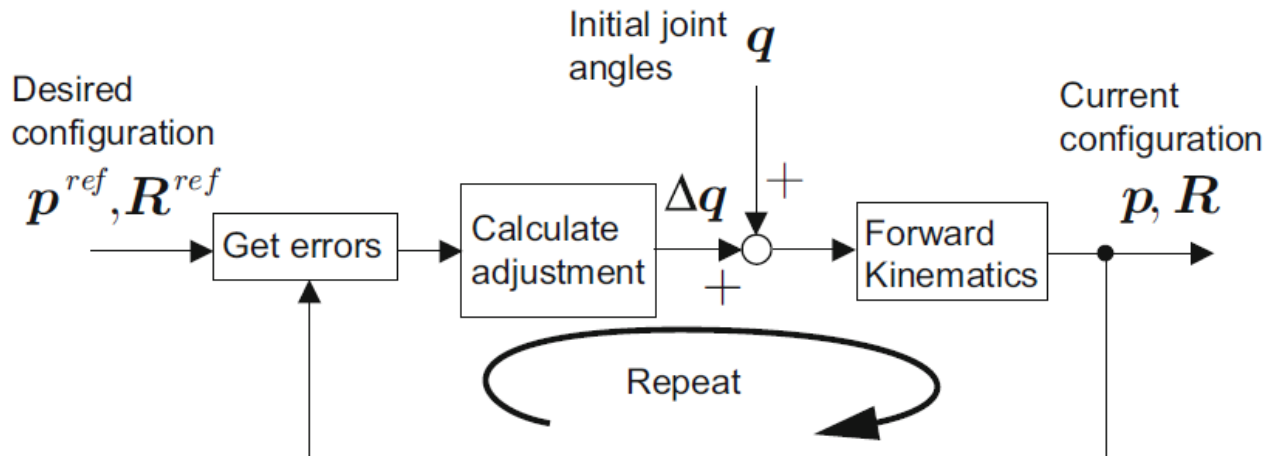
Kinematics of Humanoid Robots (III)

- Newton-Raphson method

$$\delta p = X_p(q, \delta q)$$
$$\delta \theta = X_\theta(q, \delta q).$$

$$\begin{bmatrix} \delta p \\ \delta \theta \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} \end{bmatrix} \delta q. \qquad \begin{bmatrix} \delta p \\ \delta \theta \end{bmatrix} = J \; \delta q.$$

- We calculate the adjustment with the inverse

Coefficient to stabilize the
numerical computation

$$\delta q = \lambda J^{-1} \begin{bmatrix} \delta p \\ \delta \theta \end{bmatrix}.$$

Bipedal Locomotion and ZMP

- **ZMP (Zero Moment Point)** specifies the point with respect to which dynamic reaction force at the contact of the foot with the ground does not produce any moment, i.e. the point where total inertia force equals 0 (zero).
- **ZMP is the indicator** of the stability of the robot:
  - if it is in the foot shadow (**Support Polygon**) – stable,
  - İf not – unstable.
- The shadow depends on single or double support phase.
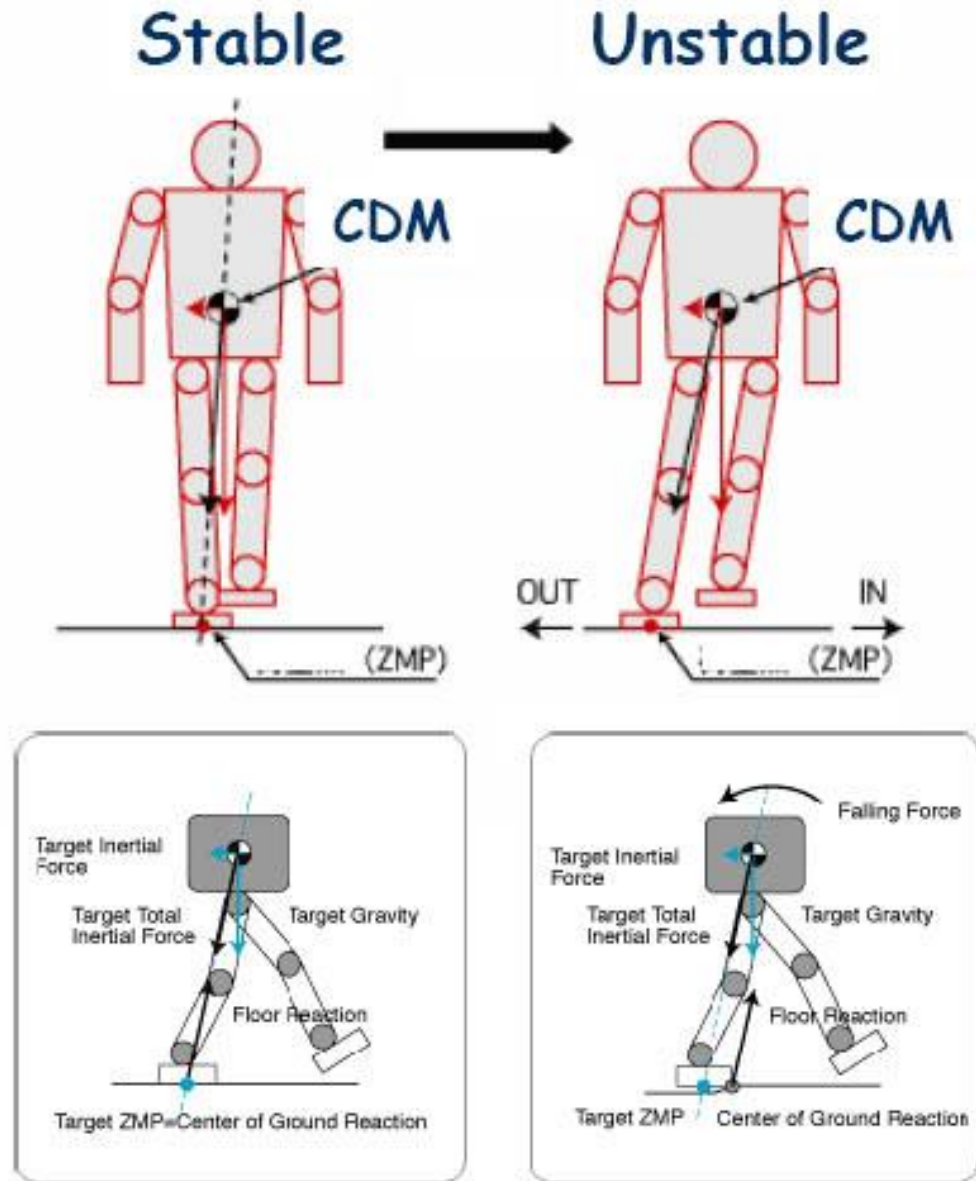
Support Polygon
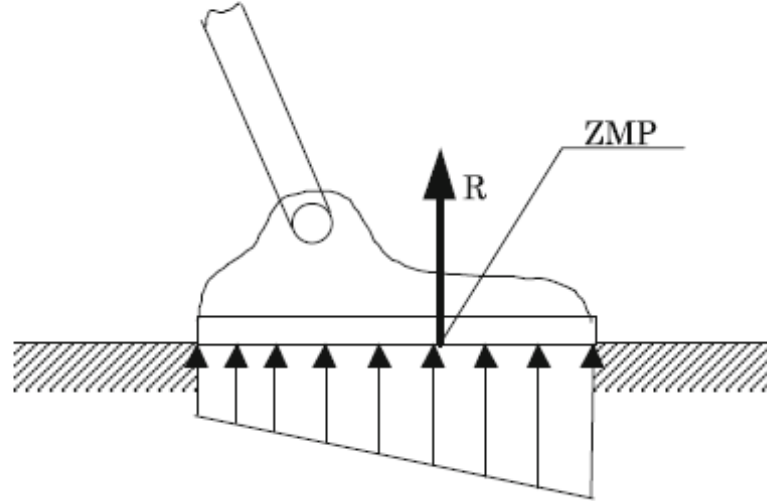
- Consider the region formed by enclosing all the contact points between the robot and the ground by using an elastic cord braid. We call this region as the **support polygon**.
- Mathematically the support polygon is defined as a convex hull, which is the smallest convex set including all contact points.
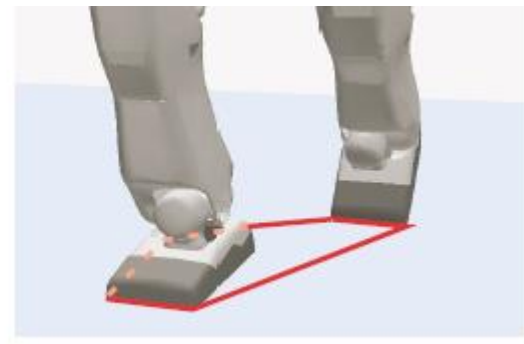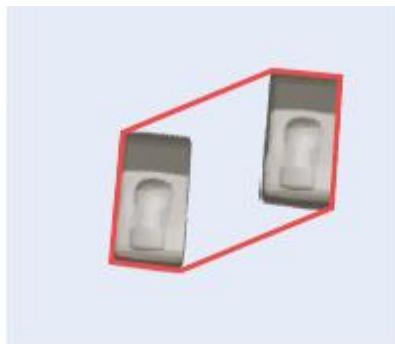
ZMP

- When a human stands on the ground, the ZMP coincides with the ground projection of CoM.

- A human can keep balance if the ground projection of CoM is included strictly inside of the support polygon.

- when a human moves dynamically the ground projection of CoM may exist outside the support polygon.

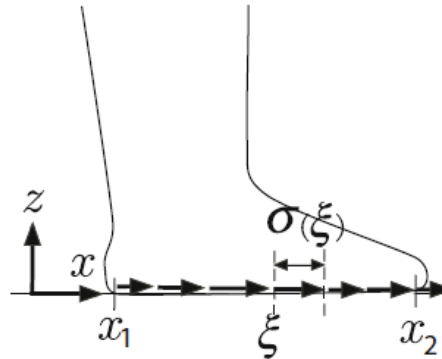- However, the ZMP never exists outside the support polygon.

CoM

ZMP

Support polygon

(a)A standing human   (b)A human in action
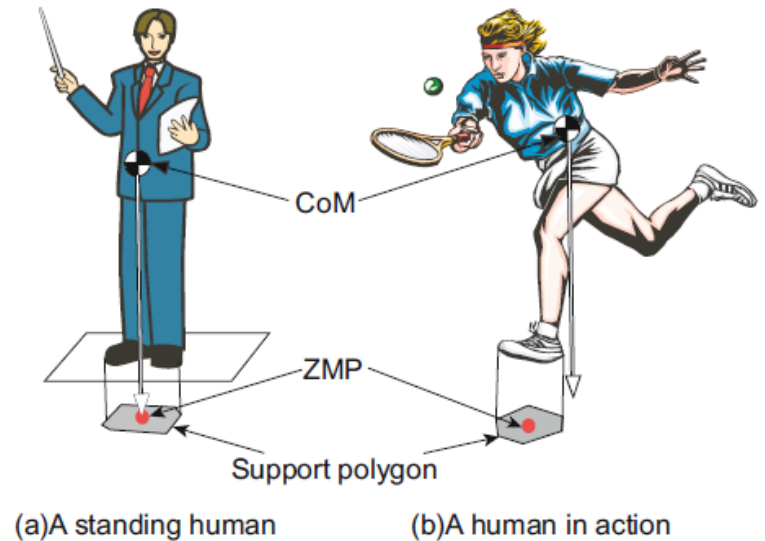
The ZMP always exists inside of the support polygon.

ZMP in 2D
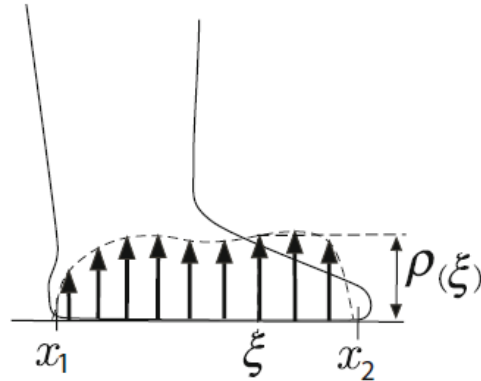
Vertical Force
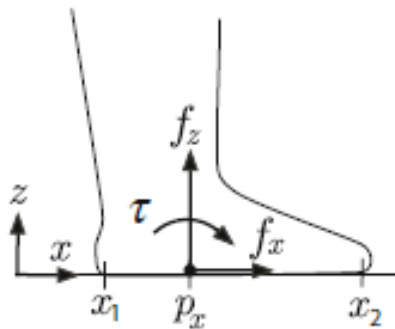

Horizontal force

The force *fx and fz and the moment τ(px) at the point px* are expressed as follows:

$$f_x = \int_{x_1}^{x_2} \sigma(\xi)d\xi$$
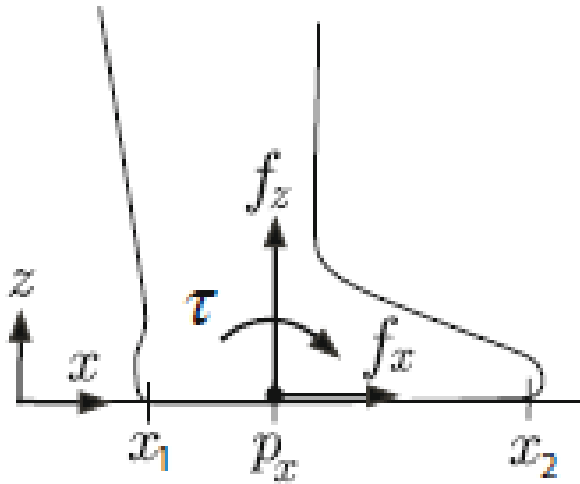
$$f_z = \int_{x_1}^{x_2} \rho(\xi)d\xi$$

$$\tau(p_x) = -\int_{x_1}^{x_2} (\xi - p_x)\rho(\xi)d\xi.$$

ZMP in 2D (2)

Considering the point where *p(x)* becomes *zero*

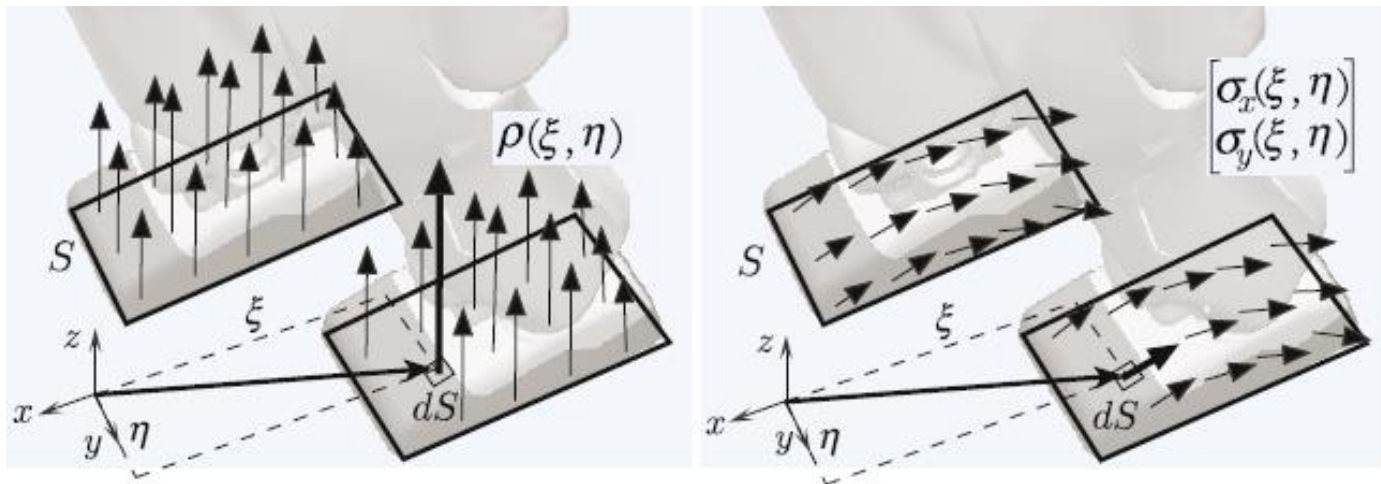$$p_x = \frac{\int_{x_1}^{x_2} \xi \rho(\xi) d\xi}{\int_{x_1}^{x_2} \rho(\xi) d\xi}.$$

Thus ***p(x)*** is the center of pressure and is the **ZMP** previously defined

ZMP in 3D

- Let $\mathbf{r}=[\xi\ \eta\ 0]^T$ the position vector defined on the ground with a vertical component $\rho=(\xi,\eta)$. The sum of the vertical component is defined as follows:

$$\boldsymbol{f}_z = \int_S \rho(\xi,\eta)\, dS,$$

ZMP in 3D (vertical component)

$\int_S$ *denotes the integration area at the contact point. The moment $\tau_{n\,(p)}$ is calculated as:*

$$\boldsymbol{\tau_n(p)} \equiv \begin{bmatrix} \tau_{nx} & \tau_{ny} & \tau_{nz} \end{bmatrix}^T$$

$$\tau_{nx} = \int_S (\eta - p_y)\rho(\xi, \eta)dS$$

$$\tau_{ny} = -\int_S (\xi - p_x)\rho(\xi, \eta)dS$$

$$\tau_{nz} = 0.$$
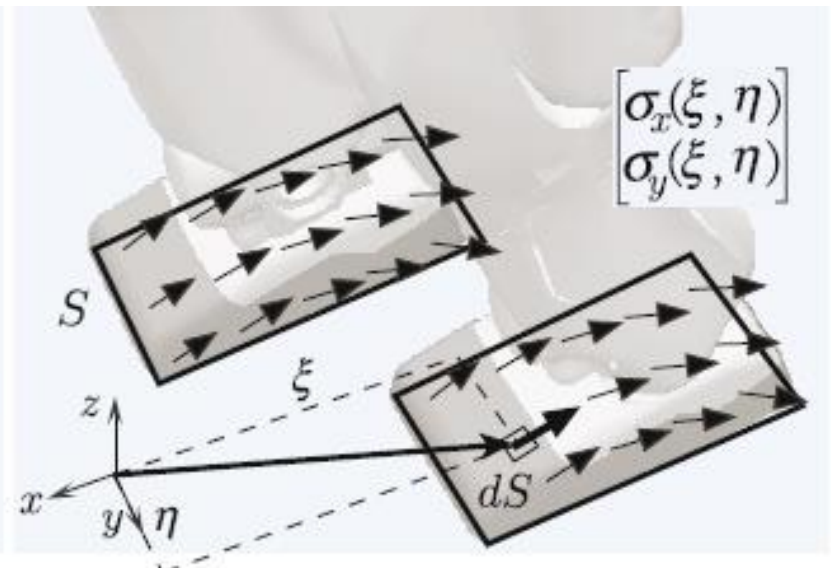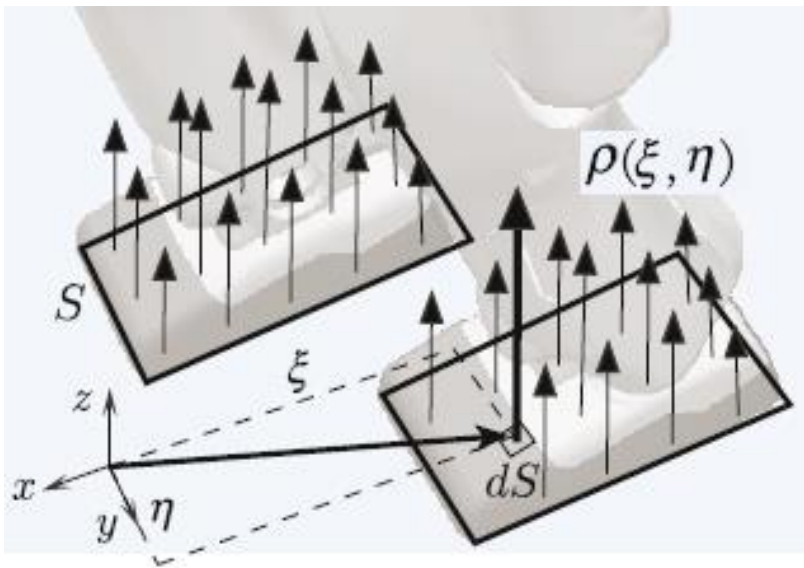
ZMP in 3D (vertical component)

Assuming as for the 2D case:

$$\tau_{nx} = 0$$

$$\tau_{ny} = 0$$

The point where the moment of the vertical component of the ground reaction force becomes zero can be expressed as:

$$p_x = \frac{\int_S \xi \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}$$

$$p_y = \frac{\int_S \eta \rho(\xi, \eta) dS}{\int_S \rho(\xi, \eta) dS}.$$

p is *the* **center of pressure or ZMP**

ZMP in 3D (horizontal component)

$$\boldsymbol{\tau}_t(\boldsymbol{p}) \equiv \begin{bmatrix} \tau_{tx} & \tau_{ty} & \tau_{tz} \end{bmatrix}^T$$

$$\tau_{tx} = 0$$

$$\tau_{ty} = 0$$

$$\tau_{tz} = \int_S \{(\xi - p_x)\sigma_y(\xi, \eta) - (\eta - p_y)\sigma_x(\xi, \eta)\} dS.$$

$$f_x = \int_S \sigma_x(\xi, \eta) dS$$

$$f_y = \int_S \sigma_y(\xi, \eta) dS.$$

These equations mean that the **horizontal ground reaction forces** generate the **vertical component** of the moment.

ZMP in 3D (horizontal component)

The ground reaction forces distributed over the surface of the sole can be replaced by the force

$$\boldsymbol{f} = [f_x \ f_y \ f_z]^T$$

and the moment :

$$\boldsymbol{\tau}_p = \boldsymbol{\tau}_n(\boldsymbol{p}) + \boldsymbol{\tau}_t(\boldsymbol{p})$$
$$= [0 \ 0 \ \tau_{tz}]^T,$$

Thus the ZMP is defined as **the point where the horizontal component of the moment of the ground reaction forces becomes zero** for 3D cases.

Force sensors

6DOF Force/Torque Sensor

p_R d
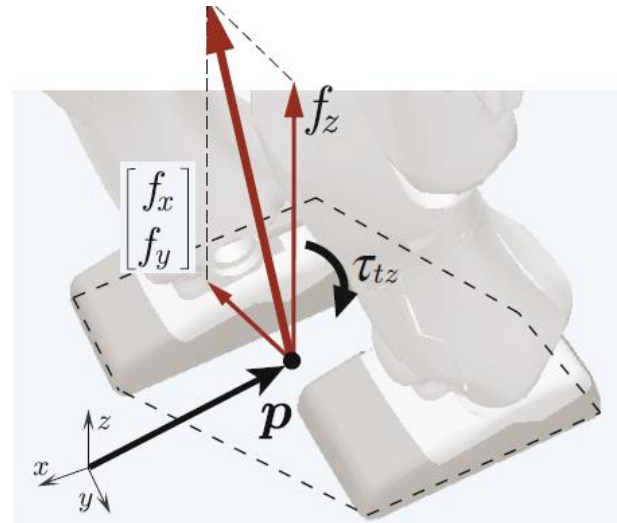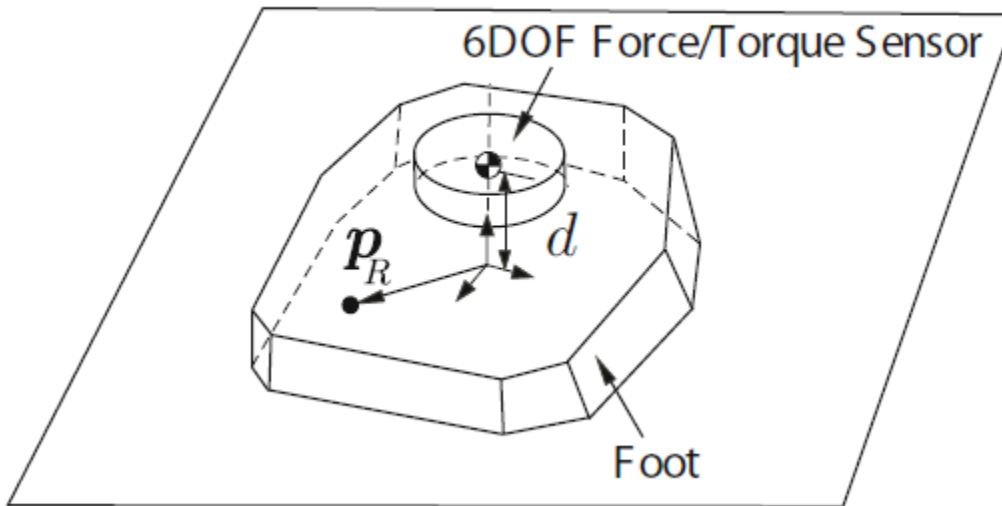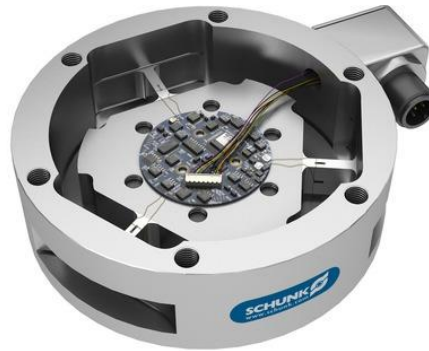
Foot

Measurement of ZMP

- **The ZMP of each foot** considering the reaction force between either one of the feet and the ground, and the ZMP considering the reaction force between both feet and the ground.

- During the double support phase, these two ZMPs becomes different.

Measurement of ZMP (II)

- Let us assume that, at the points p_j ($j = 1, ..., N$) with respect to the reference coordinate system, the forces f_j and moments τ_j are measured. Here, the moment about the point $p = [p_x\ p_y\ p_z]$ is

$$\boldsymbol{\tau}(\boldsymbol{p}) = \sum_{j=1}^{N}(\boldsymbol{p}_j - \boldsymbol{p}) \times \boldsymbol{f}_j + \boldsymbol{\tau}_j$$

$$\boldsymbol{f}_j = [f_{jx}\ f_{jy}\ f_{jz}]^T$$
$$\boldsymbol{\tau}_j = [\tau_{jx}\ \tau_{jy}\ \tau_{jz}]^T$$
$$\boldsymbol{p}_j = [p_{jx}\ p_{jy}\ p_{jz}]^T$$
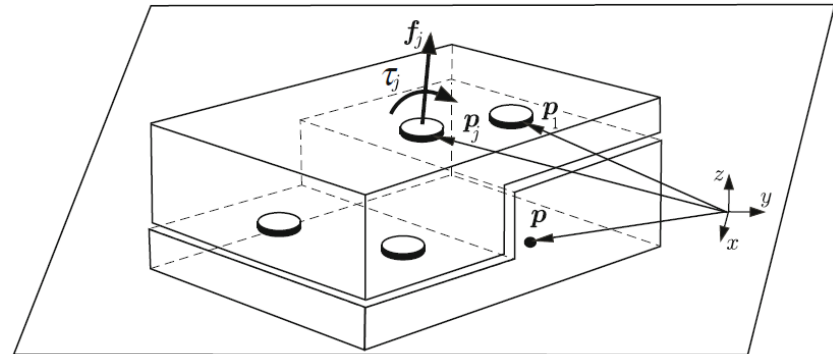
Measurement of ZMP (III)

$$\vec{\tau} = \sum_{j=1}^{N} \vec{p_j} - \vec{p} \times \vec{f_j} + \vec{\tau_j} = 0$$

Vector product

$$\begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ i & j & k \\ p_x & p_y & p_z \\ f_x & f_y & f_z \end{pmatrix}$$

$$(p_y f_z - p_z f_y)\, \hat{x}$$
$$(p_z f_x - p_x f_z)\, \hat{y}$$

Setting the *x and y conponent to zero*

$$\tau_x = 0 \quad \tau_y = 0$$

$$\tau_x = \sum_{j=1}^{N} \{[(p_{jy} - p_y)f_{jz} - (p_{jz}-p_z)f_{jy}] + \tau_{jx}\} = 0$$

$$\tau_y = \sum_{j=1}^{N} \{[(p_{jz} - p_z)f_{jx} - (p_{jx}-p_x)f_{jz}] + \tau_{jy}\} = 0$$

Basic equations for position of ZMP

Solving by p_x p_y

$$p_x = \frac{\sum_{j=1}^{N} \{-\tau_{jy} - (p_{jz} - p_z)f_{jx} + p_{jx}f_{jz}\}}{\sum_{j=1}^{N} f_{jz}}$$

$$p_y = \frac{\sum_{j=1}^{N} \{\tau_{jx} - (p_{jz} - p_z)f_{jy} + p_{jy}f_{jz}\}}{\sum_{j=1}^{N} f_{jz}}$$

Measurement of ZMP (IV)

(contact between one foot and the ground)

- The ground reaction force applied to the sole is transmitted to the sensor mount through rubber bushes and dampers.

- A 6 axis force/torque sensor is attached at the sensor mount, and the force is transmitted to the ankle of the robot through this sensor

- A 6 axis force/torque sensor is coordinated to simultaneously measure the force $f = [fx, fy, fz]$ and the moment $\tau = [\tau x \ \tau y \ \tau z]$ applied from outside the robot.

Measurement of ZMP (V)
(contact between one foot and the ground)

- Let the position of the ZMP in the right and the left foot be $\boldsymbol{p}_R$ and $\boldsymbol{p}_L$

- The position of the ZMP (for the right foot) can be obtained as:

$$p_{Rx} = (-\tau_{1y} - f_{1x}d)/f_{1z}$$
$$p_{Ry} = (\tau_{1x} - f_{1y}d)/f_{1z}$$

$$\boldsymbol{p}_R = \begin{bmatrix} p_{Rx} & p_{Ry} & p_{Rz} \end{bmatrix}^T$$
$$\boldsymbol{p}_1 = \begin{bmatrix} 0 & 0 & d \end{bmatrix}^T.$$

6DOF Force/Torque Sensor

$\boldsymbol{p}_R$

d
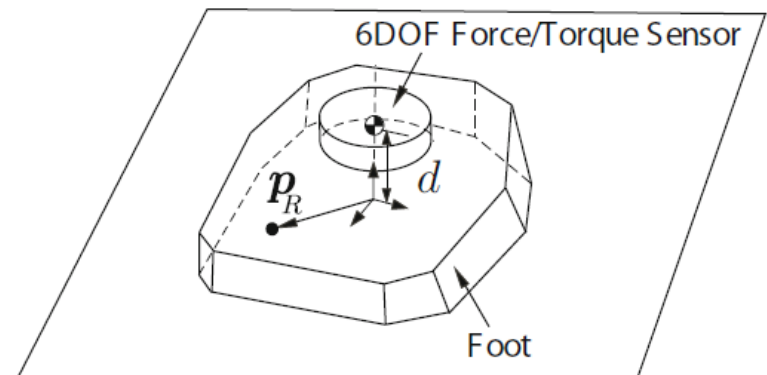
Foot

Measurement of ZMP (VI)
(both feet contact)

The position of the ZMP of each foot can be obtained as p_R and p_L. The ground reaction forces f_R and f_L are also obtained from the sensor information.

The **ZMP** in the case where both feet are **in contact with the ground** (using the basic equation for the ZMP computation) can be obtained as:

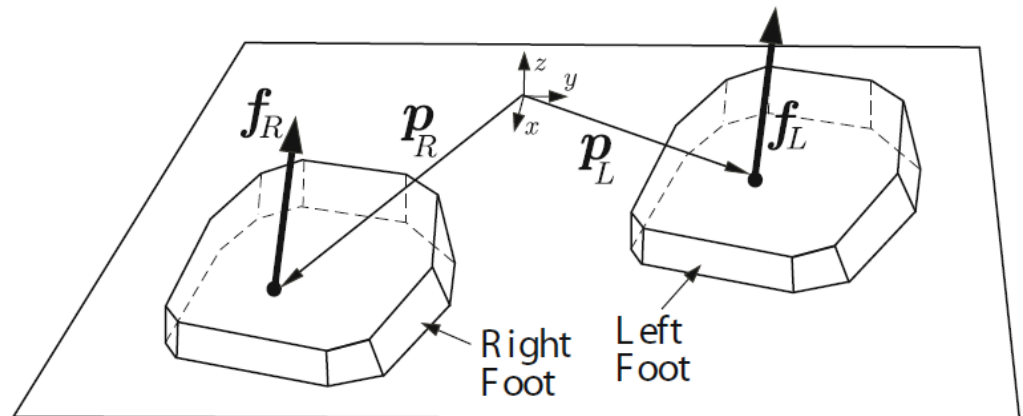$$p_x = \frac{p_{Rx} f_{Rz} + p_{Lx} f_{Lz}}{f_{Rz} + f_{Lz}}$$

$$p_y = \frac{p_{Ry} f_{Rz} + p_{Ly} f_{Lz}}{f_{Rz} + f_{Lz}}$$

$$\boldsymbol{f}_R = \begin{bmatrix} f_{Rx} & f_{Ry} & f_{Rz} \end{bmatrix}^T$$

$$\boldsymbol{f}_L = \begin{bmatrix} f_{Lx} & f_{Ly} & f_{Lz} \end{bmatrix}^T$$

$$\boldsymbol{p}_R = \begin{bmatrix} p_{Rx} & p_{Ry} & p_{Rz} \end{bmatrix}^T$$

$$\boldsymbol{p}_L = \begin{bmatrix} p_{Lx} & p_{Ly} & p_{Lz} \end{bmatrix}^T$$

Measurement of ZMP (VII)

(both feet contact)

During the single support phase, since the vertical component of the ground reaction force becomes zero, the ZMP coincides with the ZMP of the supporting foot.

$$[p_x\ p_y\ p_z]^T = \begin{cases} [p_{Rx}\ p_{Ry}\ p_{Rz}]^T & \text{for support of right foot} \\ [p_{Lx}\ p_{Ly}\ p_{Lz}]^T & \text{for support of left foot.} \end{cases}$$

Considering the balance of a humanoid robot, we can use the following equations taking both feet into account regardless of the supporting foot.

$$p_x = \frac{p_{Rx} f_{Rz} + p_{Lx} f_{Lz}}{f_{Rz} + f_{Lz}}$$

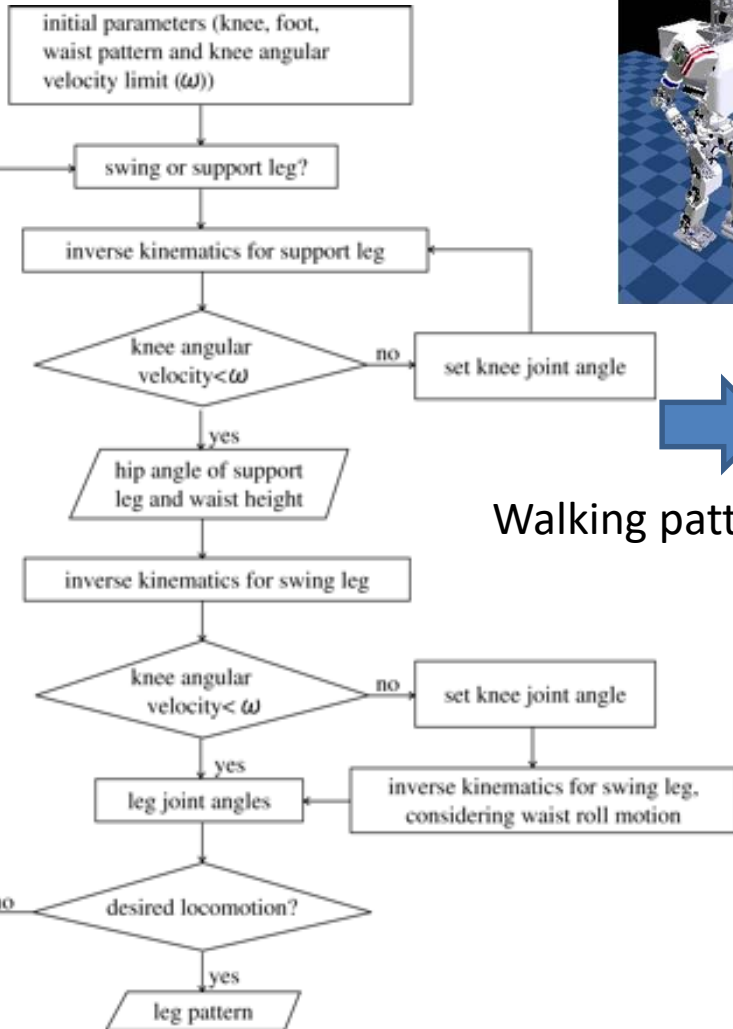$$p_y = \frac{p_{Ry} f_{Rz} + p_{Ly} f_{Lz}}{f_{Rz} + f_{Lz}}$$
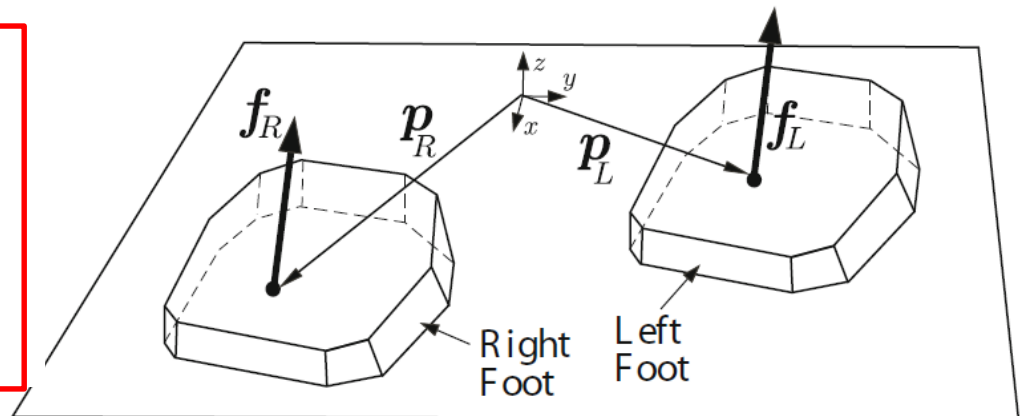
Biped walking

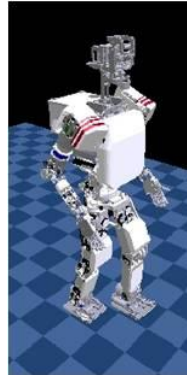- There exist two kind of walking, namely, static walking and dynamic walking
    - In "dynamic walking", there exist periods when the projection of the center of mass leaves the support polygon (human feet are too small with respect to the height of center of mass to perform static walking)
    - In "static walking", the projection of the center of mass never leaves the support polygon during the walking

Pattern Generation

- A set of time series of joint angles for desired walking is called a *walking pattern*, and to create it, it is used a *walking pattern generator*

- In an ideal situation, biped walking can be realized just by giving a walking pattern to an actual robot

- A humanoid proportion and mass distribution tends to quickly amplify the posture error to an unstable level. To suppress this, we need the second software, which modifies the walking pattern by using gyros, accelerometers, force sensors and other devices. This is called a *stabilizer*

How to realize biped walking

Pattern Generator Steps

initial parameters (knee, foot, waist pattern and knee angular velocity limit (ω))

swing or support leg?

inverse kinematics for support leg

knee angular velocity$< \omega$ — no → set knee joint angle

yes

hip angle of support leg and waist height

inverse kinematics for swing leg

knee angular velocity$< \omega$ — no → set knee joint angle

yes

leg joint angles ← inverse kinematics for swing leg, considering waist roll motion

desired locomotion? — no

yes

leg pattern

Walking pattern

→ **Stabilizer** →

Sensors feedback

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- A humanoid proportion and mass distribution tends to quickly amplify the posture error to an unstable level.
  - To suppress this, it is needed a second software, which modifies the walking pattern by using force sensors and other devices. This is called a *stabilizer*

Stabilization mechanism

We make three assumptions as coarse-graining on a humanoid robot:

1) we assume that all the mass of the robot is concentrated at its center of mass (CoM).

2) we assume that the robot has massless legs, whose tips contact the ground at single rotating joints.

3) we only consider the forward/backward and the up/down motions of the robot, neglecting lateral motion.

In other words, we assume the robot motion is constrained to the *sagittal plane* defined by the axis of walking direction and vertical axis. With these assumptions, we model a robot as a 2D **inverted pendulum**

Two dimensional inverted pendulum
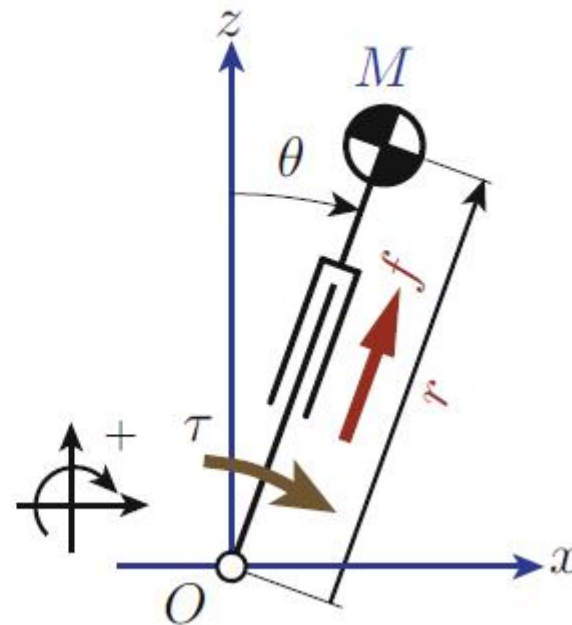
The inputs of the pendulum are the torque τ and the pivot at the kick force *f* at the prismatic joint along the leg. The dynamics of the pendulum is as follows:

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - gr\sin\theta = \tau/M$$
$$\ddot{r} - r\dot{\theta}^2 + g\cos\theta = f/M.$$

Behavor of the linear inverted pendulum

- While the vertical component of the f is canceled by gravity, the horizontal component is $M\ddot{x} = f\sin\theta.$
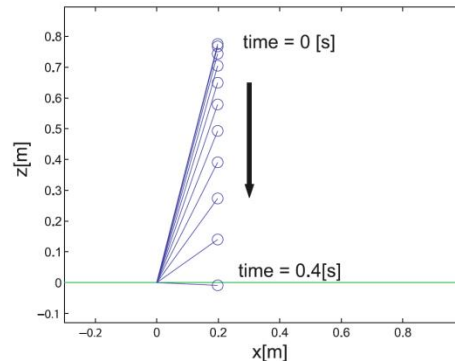
Applying different forces we get different behaviors:

(a) $f = 0$: Free fall of CoM

(b) $f = Mg\cos\theta - Mr\dot\theta^2$: Fall down with constant leg length
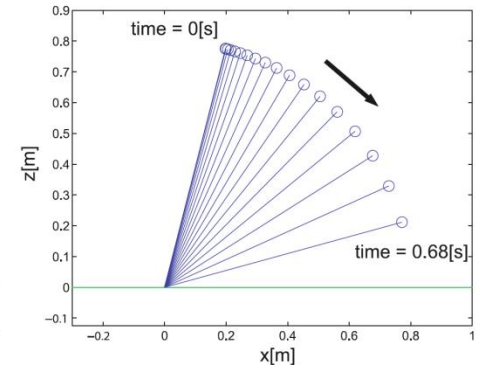
Applying the right force we obtain:

$$M\ddot{x} = \frac{Mg}{\cos\theta}\sin\theta = Mg\tan\theta = Mg\frac{x}{z}$$

$$\ddot{x} = \frac{g}{z}x.$$
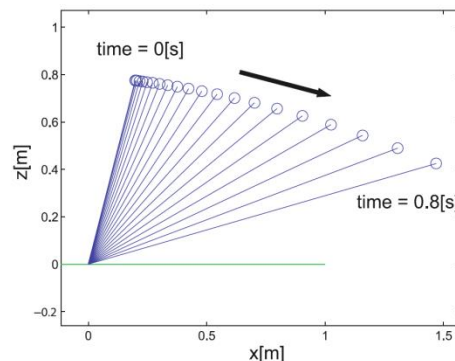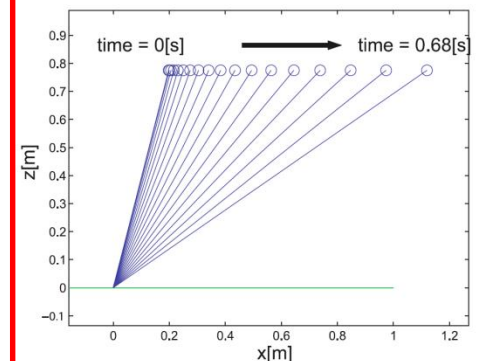

(c) $f = Mg$: Fall down and acceleration

(d) $f = Mg/\cos\theta$: CoM accelerates while keeping the initial height

ZMP and COM

- Based on knowledge of the joint trajectories or the center of mass trajectory we would like to calculate the ZMP position.

- For a desired ZMP position we could find the necessary joint trajectories or the required center of mass trajectory. The center of mass – ZMP relationship is shown below

$$\ddot{x} = \frac{g}{z_c}(x - p).$$

where, *x, z* gives the CoM of the inverted pendulum and p is the ZMP